Investigation of Archiving Techniques for Evolutionary Multi-objective Optimizers

Investigações de Técnicas de Arquivamento em Otimizadores Evolucionários Multiobjetivo

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Abstract: The optimization of multi-objective problems from the Pareto dominance viewpoint can lead to huge sets of incomparable solutions. Many heuristic techniques proposed to these problems have to deal with approximation sets that can be limited or not. Usually, a new solution generated by a heuristic is compared with other archived non-dominated solutions generated previously. Many techniques deal with limited size archives, since comparisons within unlimited archives may require significant computational effort. To maintain limited archives, solutions need to be discarded. Several techniques were proposed to deal with the problem of deciding which solutions remain in the archive and which are discarded. Previous investigations showed that those techniques might not prevent deterioration of the archives. In this study, we propose to store discarded solutions in a secondary archive and, periodically, recycle them, bringing them back to the optimization process. Three recycling techniques were investigated for three known methods. The datasets for the experiments consisted of 91 instances of discrete and continuous problems with 2, 3 and 4 objectives. The results showed that the recycling method can benefit the tested optimizers on many problem classes.

Keywords: Archiving techniques — multi-objective evolutionary algorithms — recycling techniques.

Resumo: A otimização de problemas com múltiplos objetivos sob o ponto de vista da dominância de Pareto pode levar a um conjunto de soluções incomparáveis grande. Muitas técnicas heurísticas propostas para tais problemas precisam lidar com conjuntos de aproximação que podem ser limitados ou não. Geralmente, uma nova solução gerada por uma heurística é comparada com outras soluções arquivadas previamente. Muitas técnicas trabalham com conjuntos de tamanho limitado, uma vez que comparações com arquivos ilimitados podem demandar muito esforço computacional. Para manter arquivos de tamanho limitado, algumas soluções devem ser descartadas. Diversas técnicas foram propostas para lidar com o problema de decidir quais soluções devem ser mantidas ou descartadas. Estudos anteriores mostram que essas técnicas podem não prevenir a deterioração dos arquivos. Neste trabalho, nós propomos o uso de um arquivo secundário para manter as soluções descartadas e, periodicamente, reciclá-las, trazendo-as de volta para o processo de otimização. Três técnicas de reciclagem são investigadas em três algoritmos conhecidos. O conjunto de dados utilizados nos experimentos consiste de 91 instâncias de problemas discretos e contínuos com 2, 3 e 4 objetivos. Os resultados mostraram que as técnicas de reciclagem podem beneficiar os algoritmos de otimização em várias classes de problemas.

Palavras-Chave: Técnicas de arquivamento — algoritmos evolucionários multiobjetivo — técnicas de reciclagem.

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1. Introduction

Multi-objective optimization problems are an important field of Computer Science, once real problems often have more than one objective. Compared to single objective problems, the decision-making with multiple objectives becomes more challenging. Given that solutions may be better for some objectives and worse on others, usually, there is not one optimal solution for multi-objective problems, but a set of them. Due to this fact, heuristic algorithms for multi-objective problems produce a set of solutions, which is an approximation of the optimal set. During optimization, most heuristic multi-objective algorithms keep a population or an archive of solutions that approximate the Pareto optimal set. Many important multi-objective optimizers use an external set of solutions to approximate this optimal set [1, 2, 3].

Since the comparison of solutions within an unlimited size archive may require significant processing times, many multi-objective optimizers deal with limited size archives. To keep an archive bounded, it may be necessary to discard solutions. Several techniques were proposed to deal with the problem of deciding which solutions remain in the archive and which are discarded. They are called filtering techniques and some examples are: adaptive grid archive [4], hypervolume based archive [5], adaptive ε -approximate algorithm [6] and SPEA2 [1], among others. Knowles and Corne [7] present a review about bounded archiving. Other studies showed that the filtering technique impacts on the quality of the final approximation set produced by the optimizer. The analysis presented by [8] revealed that most archiving techniques could lead to the deterioration of the archive. This occurs when, along the optimization process, the approximation set maintained by the optimizer keeps a solution that is worse than a previously discarded one, and sometimes allows an optimizer storing an entire set worse than another from a previous iteration.

To soften the deterioration problem, we propose to recycle discarded solutions bringing them back to the optimization process. We investigate three recycling methods which were tested with three evolutionary algorithms proposed for multi-objective problems: NSGAI, SPEA2, and PAES. These algorithms were chosen because they are widely used in the literature, and their archiving techniques were analyzed previously on [8] and [9]. These optimizers and their versions implemented with the recycling techniques are applied to discrete and continuous problems.

This study also extends the analysis presented by [8] with the inclusion of a theoretical and empirical investigation of the properties of four archiving techniques not investigated in that work. The filtering techniques analyzed are Non-dominated Sorting Genetic Algorithm-II [10], Strength Pareto Evolutionary Algorithm 2 [1], Adaptive Grid Archive [4], Multi-level Grid Archive [11], Ideal Archive, Distributed Archive, Distance Archive [12] and Adaptive Rectangle Archive [13].

The remainder of this paper is organized as follows. Basic concepts of multi-objective optimization problems and algorithms are presented in Section 2. Section 3 presents

some existing archiving techniques and extends the results presented by [8] with the inclusion of four archiving techniques. Section 4 presents a computational experiment with 10 archiving techniques and 7 input sequences considering 2, 3 and 4 objectives. The recycling methods are presented in Section 5. The results of a computational experiment based on 91 instances of discrete and continuous problems are presented in Section 6. Finally, some conclusions are presented in Section 7.

2. Basic Concepts

A general multi-objective minimization problem can be defined as in equation 1, where $x \in X$ is a solution, $X \subseteq \mathbb{R}^n$ is the set of feasible solutions, $f : \mathbb{R}^n \rightarrow \mathbb{R}^d$ is the function that maps x to its objective vector $f(x)$, n and d are integers. Y denotes the image of X in the objective space, $y = f(x)$, $y \in Y$.

$$\min_{x \in X} f(x) = (f_1(x), \dots, f_d(x)) \quad (1)$$

Since a solution of a multi-objective problem is mapped to a vector in the objective space, and not to a real number, the comparison of solutions in a multi-objective scenario is more complex than it is for single objective problems. Binary relations are used to define a partial order over the objective space. Some of those relations are presented in definitions 1–3 and optimal solutions are defined in 4.

Definition 1 y dominates y' , denoted by $y \prec y'$, iff $f_i(x) \leq f_i(x')$, $i = 1, \dots, d$, and $\exists k, 1 \leq k \leq d$ such that $f_k(x) < f_k(x')$. Accordingly, x dominates x' .

Definition 2 y weakly dominates y' , denoted by $y \preceq y'$, iff $f_i(x) \leq f_i(x')$, $i = 1, \dots, d$. Accordingly, x weakly dominates x' .

Definition 3 y is incomparable to y' , denoted by $y \parallel y'$, if neither $y \prec y'$ nor $y' \prec y$. Accordingly, x is said to be incomparable to x' .

Definition 4 A solution $x^* \in X$ is called efficient or Pareto optimal iff $\nexists x \in X$ such that $f(x) \prec f(x^*)$.

The set of optimal solutions is called *efficient set* or *Pareto optimal set* and is denoted by X^* . The image of X^* in the objective space is denoted by Y^* and is called *Pareto optimal front*.

Many heuristics for multi-objective optimization problems focus on delivering sets of objective vectors that are mutually incomparable. A set of mutually incomparable objective vectors is said to be a *nondominated set*. Those sets are called *approximation sets*. Sets of objective vectors can also be investigated from a dominance perspective as in definition 5.

Definition 5 Let A and B be two sets of objective vectors, $A \neq B$. If every $b \in B$ is weakly dominated by at least one element $a \in A$, then A is said to be better than B , denoted by $A \triangleright B$.

The $\triangleright$ order relation establishes that A is better than B if and only if A dominates the whole area of the objective space dominated by B . When two nondominated sets are compared, however, the $\triangleright$ relation can be not sufficient. For instance, situations in which the sets assessed have exclusive dominating areas, i.e., areas of the objective space that are dominated only by one of them. Let A and B be two different nondominated sets, such that there are areas of the objective space exclusively dominated by A and others by B . Then, neither $A \subset Y^*$ nor $B \subset Y^*$. In those and other situations, other comparison methods need to be adopted to improve the decision-making.

In this study, an algorithm for a multi-objective optimization problem is called *optimizer*. Some optimizers' designers opt to keep a limited size archive to store nondominated solutions generated during the search process. This option is made once a huge number of nondominated solutions can be generated and their maintenance in a single archive can be computationally expensive. When the number of incomparable solutions grows, the computational effort to compare a newly generated solution to those stored in the archive becomes significant when compared to other operations. The limited size archive can be the population of solutions managed by the algorithm or an external archive which works as a repository. An advantage of using a limited size archive is that less computational effort is necessary to its maintenance. A disadvantage is that "good" solutions may be lost. During the optimization process, a huge number of solutions are generated and evaluated, and as the archive is limited, many of them are discarded, even if they are not dominated by any other solution generated so far. It is desirable that the limited size archive maintains a good approximation set, according to some criteria. Let N be the maximum size of a limited archive, an optimal approximation set of bounded size N is explained in Definition 6.

Definition 6 Let A be a set of objective vectors and $|A| \leq N$, N integer. P is called an optimal approximation set of bounded size N of Y^* if there exists no set $A' \subseteq Y$, such that $|A'| \leq N$ and $A' \triangleright A$.

In the remainder of this work, the limited size archive combined with the strategy used to its maintenance is called *archiver*. A seminal paper on archiving techniques was presented by [8].

3. Archivers

Algorithm 1 shows the general template of an archiver [8], A , limited to N solutions. A_{t-1} denotes the content of A at the $t-1$ -th iteration and y a new solution to be inserted in A . If y is dominated by at least one solution of A_{t-1} (Line 1), then y is discarded. Similarly, if y dominates at least one solution of A_{t-1} , y is stored and the solutions dominated by y are removed. In case y is incomparable to each solution of A_{t-1} and $|A_{t-1}| < N$, y is stored. The problem occurs when y is

incomparable to each solution of A_{t-1} and $|A_{t-1}| = N$. In this case, a decision has to be made between to discard y or store it and discard another solution from A_{t-1} . The maintenance strategy is implemented in *filter()* which defines whether y is discarded or not and, in the last case, which solution of A_{t-1} is removed. The implementation of *filter()* defines different archiving techniques. The *nds()* function returns a set of non-dominated solutions.

Algorithm 1 Basic Archiver Template

Require: A_{t-1}, y

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1: if  $\exists x \in A_{t-1}, x \prec y$  then
2:    $A_t \leftarrow A_{t-1}$ 
3: else if  $\exists x \in A_{t-1}, y \prec x$  or  $|A_{t-1}| < N$  then
4:    $A_t \leftarrow nds(A_{t-1} \cup \{y\})$ 
5: else
6:    $A_t \leftarrow filter(A_{t-1} \cup \{y\})$ 
7: end if
```

Ensure: A_t

Many techniques for *filter()* were proposed in the literature. They were firstly analyzed by [8] and later by [9]. These analyses were concerned about desirable properties of the archivers and experimental results regarding quality indicators. The investigated properties, as presented by [8], are revised in Section 3.1 and the archivers in Section 3.2.

3.1 Properties and Convergence

For the sake of simplification, solutions are referred as objective vectors or merely as points.

Property 1 *Diversifies*. An archiver is said to be efficiency preserving when during updating A_t to A_{t+1} , $|A_t| = N$, only points of the dominating region of A_t are allowed to be accepted to A_{t+1} . If an archiver does not have this property, then it is said to diversify [14].

Property 2 *Monotone*. An archiver is called monotone if there exists no possibility to one solution from A_t dominates any solution from A_u if $u > t$. $\forall x \in A_u, \forall x' \in A_t, x \prec x' \Rightarrow u > t$.

Property 3 $\triangleright$ -Monotone. An archiver is said $\triangleleft$ -monotone if and only if $\forall A_t, A_u \subset \Omega, A_t \triangleright A_u \Rightarrow t > u$. This property is similar to monotone, but with $\triangleright$ relation between sets, not solutions. Obviously, if any archiver is not $\triangleright$ -monotone, it will not be monotone.

Property 4 $\subseteq Y^*$. An archiver has this property if it contains only optimal solutions from the processed input sequence, i.e., $\forall t, A_t \subseteq Y_t^*$.

Property 5 *Limit-stable*. Let P be a set of solutions, and X an archiver. If inserting indefinitely points from P into X makes X to converge to a stable set X_t in finite time, then X has the limit-stable property.

Property 6 *Limit-optimal.* Let P be any set of solutions, and X an archiver. If inserting indefinitely points from P into X makes X converges to an optimal approximation set (see Definition 6) X_t in finite time, then X has the limit-optimal property.

3.2 Methods and Selection Algorithms

Unbounded Archiver. The archive is not bounded. Given an input sequence of points, the unbounded archiver leads to the Pareto Front of this sequence.

Dominating Archiver. In this technique, a new point y is accepted only if $|A_{t-1}| < N$ or if y dominates at least one solution of A_{t-1} . Its $filter(A_{t-1} \cup \{y\})$ function returns A_{t-1} . Although this archiver does not diversify, it does not ensure the $\subseteq Y^*$ property.

NSGA-II. The archiving technique named NSGA-II in this study comes from the selection mechanism of the multi-objective genetic algorithm proposed by [10]. Since the population of that genetic algorithm is limited, the selection mechanism defines which solutions go to the next generation. Here, the $filter$ procedure consists in discarding the point from $\{y\} \cup A_{t-1}$ in the most crowded region. To measure how much crowd is the region of a point, the *crowding distance* is computed. The method to compute the crowding distance is presented in Algorithm 2. The list of solutions is sorted by the first objective, then by the second and so on. The crowding distance of the i -th solution is computed from the distance from i to the immediately previous ($i - 1$) and next ($i + 1$) solutions. The solution with the smallest crowding distance is discarded. As shown by [8], the NSGAII archiving technique cannot guarantee the convergence of the archiver to an optimal non-dominated set. The archiver can deteriorate and $\triangleleft$ -deteriorate (it is not monotone nor $\triangleleft$ -monotone).

Algorithm 2 Crowding Distance Assignment

Require: L

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1:  $l \leftarrow |L|$ 
2: for each solution  $L(i)$  do
3:    $L(i).distance \leftarrow 0$ 
4: end for
5: for each objective  $m$  do
6:   Sort  $L$  according to objective  $m$ 
7:    $L(1) \leftarrow L(l) \leftarrow \infty$ 
8:   for  $i \leftarrow 2$  to  $(l - 1)$  do
9:      $L(i).distance \leftarrow L(i).distance + (L(i + 1).m +$ 
        $L(i - 1).m)$ 
10:   end for
11: end for

```

Ensure: L

SPEA2. The archiving technique named SPEA2 also comes from a genetic algorithm proposed for multi-objective problems presented by [1]. In this case, the $filter()$ function calculates first the distance between every pair of points in A , associates the minimum distance computed for each point

with that point and discards the point associated with the overall minimum distance. Ties are broken by considering the second smallest distance, and so on. As the NSGAII archiver, its content may $\triangleleft$ -deteriorate [8].

Adaptive Grid Archiver (AGA). This archiver was proposed by [5]. The objective vectors are split into a grid. Each cell of the grid may contain many solutions. $filter()$ removes one solution, chosen at random, from the most crowded cell, excluding a minimum or maximum on any objective. It is not monotone nor $\triangleleft$ -monotone [8].

Hypervolume Archiver (AA). This archiver technique was proposed by [5]. It aims at maximizing the hypervolume of the approximation set. Its $filter()$ function returns the subset of $A_{t-1} \cup \{y\}$, limited to size N , with the maximum hypervolume. As the hypervolume calculation is exponential on the number of objectives, AA may spend more time than the other techniques. Besides, this greedy approach does not ensure that the hypervolume will be maximum at the end of the input sequence, as shown by [15].

Multi-level Grid Archiving (MGA). Proposed by [11], the MGA uses a hierarchical family of boxes of different levels over the objective space. Its $filter()$ function finds the smallest level b where at least one solution from the boxes at level b is dominated. If the point to be inserted is one of the dominated points at level b , then it is rejected. Otherwise, a random point is removed. MGA can deteriorate, as shown by [8], but it is $\triangleleft$ -monotone.

Ideal Archiver (Ideal). This archiver was proposed by [12] to control convergence in particle swarm optimization. It first calculates an ideal point, which is the vector with the best values for each objective. After that, the archiver discards the point with higher Euclidean distance from the ideal point. The main idea behind this technique is to converge to only one area of the objective space.

Distributed Archiver (Distributed). This archiver also uses the ideal point to guide the $filter()$ function. However, its convergence is not directed to only one point, but to $d + 1$ points – the ideal point and one reference point for each dimension. The reference points are the closest points to each dimension, defining $d + 1$ regions of the objective space (one next to ideal point and d to each reference point). Each region stores $(N + 1)/(D + 1)$ points, i.e., the number of points divided by the number of regions. For each dimension, each set consists of the $(N + 1)/(D + 1)$ closest points to each reference point. The point farthest from any reference point is discarded and A_t is defined as the union of the reference sets. This scheme does not guarantee the archiver to be $\triangleleft$ -monotone, and consequently, it is not monotone and has not the $\subseteq Y^*$ property.

Distance to Reference Points Archiver (Distance). Also proposed by [12], it works similarly to the Distributed Archiver, first calculating the same reference points. However, it does not limit the region size to $(N + 1)/(D + 1)$. For each point in $A_{t-1} \cup \{y\}$ the smaller distance from each reference point is calculated. The point with the higher value is discarded.

Distance Archiver only has the Diversify property, out of the six presented before.

Adaptive Rectangle Archiver (ARA). This one does not follow the pattern of Algorithm 1 and it is not limited to N points. ARA is based on two steps: to define a region adaptively and divide it into rectangles, such that each rectangle only contains one point [13]. The region is defined by two vectors $a^{(min)}$, which is the same Ideal Point proposed by [12], and $a^{(max)}$, the vector with the worst objective values for each dimension in A . ARA uses the E-dominance concept to decide in which rectangle a point is placed. E-dominance concept is similar to ε -dominance, except from $E \in \mathbb{R}^d$, while $\varepsilon \in \mathbb{R}$. So, the lowest the value of E_i , more finely and accurately the dimension of the i -th objective is split. Moreover, the archiver saves the same reference points of the Distributed Archiver, keeping a set of d solutions with the best values of each objective found so far. The archiver has the six desirable properties. However, due to its flexible structure, it has some problems that are presented in the next section.

Table 1 shows a summary of the properties of each archiver and extends the table presented by [8]. Whereas most of the archivers are not monotone, and only two of them have the $\subseteq Y^*$ property (one is the unbounded archiver), the main idea of this work emerges: to recycle discarded solutions and bring them back to the optimization process.

Table 1. Archivers and their properties

Archiver	Div.	Mon.	$\triangleleft$ -Mon.	$\subseteq Y^*$	L-St	L-Op
Unbounded	✓	✓	✓	✓	✓	✓
Dominating	✗	✓	✓	✗	✓	✓
NSGA-II	✓	✗	✗	✗	✗	✗
SPEA2	✓	✗	✗	✗	✗	✗
AGA	✓	✗	✗	✗	✗	✗
AA	✓	✗	✓	✗	✓	✓
MGA	✓	✗	✓	✗	✓	✓
Ideal	✓	✗	✓	✗	✓	✓
Distributed	✓	✗	✗	✗	✗	✗
Distance	✓	✗	✗	✗	✗	✗
ARA	✓	✓	✓	✓	✓	✓

4. Experiments with Fixed Input Sequences of Points

We analyzed the quality of the sets generated by each archiver presented in Section 3.2 for fixed input sequences. We tested each archiver with different input files, each of them consisting of a set of points (objective vectors). The experiment consisted in adding each point of the input file iteratively to a limited size archive, initially empty. The quality of the set stored by each technique was assessed according to the quality indicators presented in Section 2 and the number of optimal solutions stored at the end of the process. This experiment extended the one reported by [8] with the inclusion of 3 new input files and 4 archivers. The new input files are different from those presented by [8] with regard to dimension, scale and structure. They were created to investigate specific problems

of the archivers proposed by [12] and [13]. The value of each component of vector E , used by ARA, was $E_i = \pi/100, \forall i = 1, \dots, d$.

The archivers were tested with 7 input files: (1) *smallPF-2d-10000*, (2) *smallPF-3d-10000*, (3) *2000-3d*, (4) *10000-4d*, (5) *clustered-2d-900*, (6) *1to2-2d-2000* and (7) *1000-clustered-3D*. Sequences 1, 2, 5 and 6 were provided by [8].

Files *smallPF-2d-10000* and *smallPF-3d-10000* have 10,000 points with 2 and 3 objectives. Their optimal sets contain 970 and 1,335 points, respectively. They were generated from a real optimizer solving a problem. The *1to2-2d-2000* file contains a sequence of 2,000 incomparable points, from (1,2) to (2,1), with the first objective increasing while the second decreases. Files *2000-3d* and *10000-4d* contain 2,000 and 4,000 points with 3 and 4 objectives. They were created by a specific algorithm that generates random points with objective values between 0 and 1. *1000-clustered-3D* and *clustered-2d-900* are files containing, respectively, 1000 and 900 clustered points with 3 and 2 objectives. These files were used to investigate some properties of ARA. The archives were limited to $N = 100$, except the one obtained with ARA, which is not limited due to the archiver's structure.

Table 2 shows the number of optimal solutions saved by each archiver. Line 1 shows the references to the files. Line *Unbounded* shows the size of the optimal set of the points contained in each file. Dominating and MGA finished the process with archives containing only optimal solutions for all input sequences. AA, Ideal, Distributed and Distance failed to find optimal solutions only in one case, losing 1, 2, 23 and 14 optimal solutions, respectively. The NSGA-II archiver presented a weak result on the 4-dimension file, which showed to be also the hardest instance for SPEA2, Ideal, Distributed and Distance. The average number of optimal points for each archiver were: 100 to Dominating and MGA, 99.8 for AA, 99.6 for Ideal, 97.2 for Distance, 96.2 for AGA, 95.4 for Distributed, 94.4 for SPEA2, 83.4 for NSGA-II and 45.4 for ARA.

Except for the 4-dimensional file, ARA managed to keep significantly fewer solutions than the other techniques. The results show that ARA could not keep a representative number of solutions when tested to two-dimensional vectors. Besides, when tested to clustered points, the archiver presented the same problem, even if the vectors had a higher dimension.

We applied the hypervolume [16], denoted $HYP()$, and the unary additive ε [17], denoted by $\varepsilon_{add}()$ to the final sets. Tables 3 and 4 show the results of the hypervolume and ε -add indicators, respectively. The hypervolume is maximized and the ε -add minimized. The results presented in table 3 are percentages of the Pareto Front hypervolume. The archivers were ranked according to average results and presented in this order. The values on Table 4 have to be multiplied by 10^4 on files 1 and 2, by 10^2 on files 3, 4 and 6, and by 10^3 on file 5.

Although [15] showed that being greedy did not ensure that the hypervolume is maximum at the end of the input sequence, except from *1to2-2d-2000* (instance 6), AA presented

Table 2. Number of optimal points stored by each archiver technique.

Archiver	1	2	3	4	5	6	7
Unbounded	970	1,335	500	1000	300	2,000	93
Dominating	100	100	100	100	100	100	93
NSGA-II	94	99	97	27	100	100	93
SPEA2	94	99	99	80	100	100	93
AGA	91	95	100	95	100	100	93
AA	99	100	100	100	100	100	93
MGA	100	100	100	100	100	100	93
Ideal	100	100	100	98	100	100	93
Distributed	100	100	100	77	100	100	93
Distance	100	100	100	86	100	100	93
ARA	8	47	39	131	2	2	3

Table 3. Hypervolume results for each archiver.

Archiver	1	2	3	4	5	6	7
Unbounded	100	100	100	100	100	100	100
AA	99.99	99.99	99.98	99.92	99.97	99.79	100
SPEA2	99.99	99.99	99.89	94.03	99.96	89.70	100
NSGA-II	99.99	99.99	99.44	77.17	99.94	99.85	100
AGA	99.99	99.99	99.45	97.70	99.43	99.38	100
MGA	99.97	99.85	99.95	99.39	99.90	99.38	100
Dominating	99.99	99.99	99.41	99.04	99.41	59.94	100
ARA	99.97	99.99	97.79	99.18	4.62	85.72	82.73
Ideal	98.97	98.80	99.97	99.79	93.07	59.94	100
Distributed	99.99	99.97	99.20	95.87	85.12	87.08	100
Distance	99.97	99.92	90.28	89.58	34.67	87.08	100

the best results for all instances. The Dominating archiver, obviously, kept the first 100 vectors until the end of the experiment, which resulted in a poor output, considering both quality indicators.

As AA verifies the best set (regarding hypervolume) each iteration, it is natural to present the best performance this indicator. The archivers proposed by [12] did not go well for this indicator, and they were ranked in the last positions. A conclusion is that concentrating the points is not a good strategy if one wishes to maximize the hypervolume, once this indicator aims at measuring how good is the distribution of the points of a set.

MGA and AA had the best ε indicator results. Despite the fact that AA has higher time complexity than the other archivers, it can be considered the winner of these experiments for the two indicators. Although the MGA performed well for the ε -add, it was ranked in the 5th place for the previous indicator. Distance Archiver had poor results for this indicator, as well. Its strategy may lead the archive to $\triangleleft$ -deteriorate easily, as shown by [9].

Table 5 presents the times spent by each archiving technique to process the input sequences. AA and ARA were the most time-consuming archivers, in particular for the four-dimensional instance, which indicates that if they were used within a heuristic algorithm, there would be an excessive spending to deal with archiving. Dominating archiver was the fastest one, as expected, once it is not allowed to diversify and just refuses non-dominated points when out of capacity.

Table 4. ε -add indicator results for each archiver.

Archiver	1	2	3	4	5	6	7
MGA	0.60	4.40	0.65	5.25	1.85	1.55	0
AA	0.60	23.1	1.04	3.86	1.67	16.08	0
NSGA-II	0.60	9.20	4.99	25.34	1.75	0.70	0
SPEA2	0.70	6.60	3.33	21.60	2.03	42.42	0
AGA	4.50	20.5	5.76	12.18	5.09	10.20	0
Ideal	26.0	22.9	1.84	05.82	243.60	95.05	0
Dominating	4.50	26.1	5.76	11.69	5.09	95.05	0
ARA	8.30	8.00	7.37	08.02	496.23	49.98	0.12
Distributed	22.6	24.6	4.28	12.82	122.76	47.52	0
Distance	56.7	65.5	9.69	12.82	411.99	47.52	0

Table 5. Time spent (seconds) by each archiving technique.

Archiver	1	2	3	4	5	6	7
Dominating	0.03	0.04	0.00	0.03	0.00	0.01	0.01
NSGA-II	13.00	15.09	3.11	17.60	0.90	2.8	1.28
SPEA2	12.93	14.88	2.71	14.65	1.11	2.54	1.40
AGA	0.11	0.14	0.03	0.15	0.02	0.08	0.01
AA	21.97	32.51	6.13	77.60	2.58	5.60	1.53
MGA	3.39	3.74	0.58	1.78	0.58	1.66	0.01
Ideal	0.66	0.76	0.12	0.74	0.08	0.21	0.04
Distance	0.83	1.06	0.18	1.05	0.12	0.36	0.04
Distributed	12.91	11.79	1.95	7.33	2.57	8.08	0.05
ARA	1.30	30.51	3.75	141.33	0.02	0.12	0.07

5. Recycling Solutions

Based on the analysis shown in Section 3, this work proposes a deeper investigation on how helpful discarded solutions can be within an optimization process. The method consists of two steps. First, a recycle bin is created to store solutions rejected by the archiving technique used by an optimization algorithm. Including solutions in the recycle bin requires $O(1)$ processing time. No verification of Pareto dominance is done in this step. Periodically, the optimizer verifies the content of this bin and recovers good solutions. The goal is to improve the quality of the approximation set (population or external archive) along the optimization process. In this study, the population or external archive is notated as P and the recycle bin as Bin . Since the archiving techniques may deteriorate or not $\subseteq Y^*$, there can be solutions in P dominated by solutions in Bin . Therefore, this study proposes bringing solutions from Bin to P , $|P| = N$.

Periodically, the optimizer selects N solutions from $\{P \cup nds(Bin)\}$ to maintain in P , where $nds(Bin)$ stands for non-dominated solutions from the recycling bin. There may be many ways of selecting these solutions. Three recycling methods are proposed and investigated in this study: (i) random selection, (ii) crowding distance selection and (iii) deterioration verification. Thereby, four scenarios are investigated for each optimizer: no recycling and one for each recycling method. The remainder of this section presents the recycling methods, the optimizers where they were implemented and the problems used in the experiments.

5.1 Recycling Methods

Random selection, crowding distance and deterioration verification recycling methods are notated as *random*, *crowdd* and *verif*, respectively. *Clean* denotes the version of the optimizer without recycling. Algorithm 3 illustrates the use of a recycling method within an optimizer. The optimization process is executed in the *process()* procedure, for instance, the steps of a Genetic Algorithm. Solutions discarded by the archiving technique are sent to the recycling bin in Line 3. Recycling takes place in Lines 4-6. Every K iterations dominated solutions in *Bin* are discarded and P_{i+1} is updated by a recycling method. The proposed methods are described in the following where *Bin* stands for the recycling bin after dominated solutions are removed.

Algorithm 3 Recycling

Require: P_i, Bin

```

1: while not Stop condition do
2:    $P_{i+1} \leftarrow process(P_i)$ 
3:    $Bin \leftarrow Bin \cup (P_i - P_{i+1})$ 
4:   if  $i$  is multiple of  $K$  then
5:      $Bin \leftarrow nds(Bin)$ 
6:      $P_{i+1} \leftarrow recycle(P_i, Bin)$ 
7:   end if
8: end while

```

Ensure: P_{i+1}, Bin

Random Selection: This method selects N random points from $\{P_i \cup Bin\}$.

Crowding Distance Selection: The best N solutions according to crowding distance are selected. This metric was proposed by [10] in the NSGA-II algorithm to ensure diversity of the population. It assigns low priorities to solutions located in crowded regions of the objective space (see Algorithm 2).

Deterioration Verification: Given P and *Bin*, the method verifies if each solution $P_i \in P$ is dominated by some solution in *Bin*, i.e., $\exists P_i \in P, \exists Bin_j \in Bin$ such that $Bin_j \prec P_i$. If so, the dominated solution in P is replaced by Bin_j . In case of more than one solution in *Bin* dominate P_i , then P_i is replaced by one of them selected randomly.

6. Computational Experiments

We investigated whether the recycling methods can improve the performance of some evolutionary optimizers concerning their outcomes. We did not focus on comparing distinct optimizers to specific problems. To evaluate the potential of the proposed recycling techniques, three well-known optimizers, NSGA-II [10], SPEA2 [1] and PAES [2], were tested on the benchmark problems presented in section 6.1. The SPEA2, NSGA-II and PAES implementations were obtained from the jMetal framework [18]. The experiments were executed on a PC with an Intel Core i7-3632QM processor and 2.2 GHz of RAM, which ran Ubuntu 14.04 LTS 64Bits.

The stop criterion of the NSGA-II and the SPEA2 was 350

iterations. Once the PAES algorithm generates only one solution per iteration, its stop criterion was $350 * |N|$, where $|N|$ is the size of the population of the NSGA-II and the SPEA2, $|N| = 50$. Default jMetal's mutation and crossover operations were used by the NSGA-II and the SPEA2. Recycling was executed every 50 iterations.

Each experiment (optimizer + archiving technique $\rightarrow$ problem) was executed 30 times, independently, with integer seeds for the random number generator from the range of $[1, 30]$.

Hypervolume and ε -additive quality indicators were used to assess the approximation sets generated by each optimizer. The quality indicators were computed every 50 iterations of each execution. However, for the sake of space, we report the results computed for the final approximation sets, i.e., those obtained at the end of each run. If necessary, complete results are reported to clarify some behavior of a specific optimizer/problem. The Kruskal-Wallis (KW) test [19], at a significance level of 0.05, was employed to detect differences in the behaviors of the optimizers. The reference set needed to compute the ε -add indicator consisted of the nondominated solutions from the 30 final approximation sets produced by each optimizer + archiving technique $\rightarrow$ problem. The reference point needed to compute the hypervolume indicator consisted of the worst values found for each objective. Thus, the reference point was dominated by all points of the reference set.

Section 6.1 presents the problems used in the experiments. Sections 6.2, 6.3 and 6.4 present the results of the experiments for the NSGA-II, SPEA2 and PAES, respectively.

6.1 Problems

Discrete and continuous benchmark problems were used in the experiments. The problems are described as follows.

6.1.1 Knapsack Problem

Given a set with k items, $L = \{l_1, l_2, \dots, l_k\}$, each item with a profit p_i and a weight w_i , $i = 1, \dots, k$, and a knapsack with capacity W , the Knapsack Problem (KP) consists in selecting the items which maximize the total collected profit, so that the maximum weight W is not exceeded. In the multi-objective version of the KP (mKP), d profits and d weights are associated with each item. There are d knapsack capacities, $W_1, \dots, W_d$. The mKP formulation is given in (2)–(4).

$$\max f(x) = \sum_{i=1}^k x_i p_1^i, \sum_{i=1}^k x_i p_2^i, \dots, \sum_{i=1}^k x_i p_d^i \quad (2)$$

subject to

$$\sum_{i=1}^k x_i w_j^i \leq W_j \quad \forall j \in \mathbb{N}, j = 1, \dots, d \quad (3)$$

$$x_i \in \{0, 1\} \quad (4)$$

The mKP instances were generated with the random generator from PISA [20] which was implemented on the jMetal framework. Instances were generated with 100, 200, 500 and 750 items, 2, 3 and 4 objectives. The seed computed for the random generator was given by $seed = 10 * k + d$, where k is the number of items and d the number of objectives. The mKP instances are named KSk- d .

6.1.2 Quadratic Assignment Problem

The quadratic assignment problem (QAP), as proposed by [21], consists in assigning n industrial plants to n locations, such that the cost of the flow of commodities supplied by one facility to another is minimized. The m objective functions of a multi-objective QAP (mQAP), as presented by [22], are formulated in (5), where the k -th objective function is given in (6), $k = 1, \dots, m$.

$$\min C(\phi) = \{C^1(\phi), C^2(\phi), \dots, C^m(\phi)\} \quad (5)$$

$$C^k(\phi) = \sum_{i=1}^n \sum_{j=1}^n a_{ij} b_{\phi_i}^k \phi_j \quad (6)$$

The mQAP instances were generated with the generator proposed by [22]. Instances were generated with 10, 20, 30 and 40 facilities and 2, 3 and 4 objectives. The mQAP instances are named $mqapnndz$, where nn is the number of facilities, d the number of objectives and z the instance identification.

6.1.3 Continuous Problems

The benchmark continuous problems used in the experiments were: WFG [23], LZ09 [24], DTLZ [25], ZDT [26], Viennet2 and Viennet3, by [27], Fonseca [28] and Kursawe [29]. WFG problems were proposed to create a toolkit to generate multi-objective optimization problems with multiple and controlled features. On this toolkit, it is possible to add intended complexities to the problem, such as non-separability or multi-modality. Furthermore, it is possible to control the nature of the Pareto front, making it as linear, convex, concave, convex/concave and disconnected. The work of [23] provided 9 problems based on this toolkit, called WFG1–9. All those problems were tested in the experiments reported in this study. LZ09 problems were proposed by [24] to test variants of some multi-objective evolutionary algorithms.

6.2 Recycling with the NSGA-II

The NSGA-II (Non-dominating Sorting Genetic Algorithm II) is a multi-objective genetic algorithm proposed by [10]. It is one of most studied multi-objective algorithms in the literature. Initially a population, P_1 , with N individuals, is created and evaluated. Then, every new iteration genetic operators are applied to the individuals of the current population generating a new population, Q . The population of the i -th iteration, P_{i+1} , is obtained from $P_i \cup Q$. First, $P_i \cup Q$ is divided into r subsets, F_k , $k = 1, \dots, r$, called fronts, such that F_1 contains the nondominated solutions of $P_i \cup Q$ and every element of

F_k , $k = 2, \dots, r$, is dominated by at least one element of F_{k-1} . Fronts F_k are added to P_{i+1} in ascending order of their indexes up to reaching N individuals. In case P_{i+1} is not completed after the insertion of the k -th front and the k -th front does not fit completely in P_{i+1} , the solutions from F_k with the largest values of crowding distance (Algorithm 2) complete the new population.

6.2.1 mKP

Table 6 shows the results of the NSGA-II and its variants with recycling techniques for mKP instances. The table shows the average hypervolume and ε -add, computed from the output of 30 independent executions of each NSGA-II variant. The columns show the instance name, the factor by which the hypervolume values have to be multiplied, and the results for each variant. Bold values indicate the best values for each quality indicator.

When compared with the clean version, the *verif* variant presented the best hypervolume results for 8 from the 12 instances tested, and the best ε -add values for 7 instances. The clean version was better than the *crowdd* and the random version for 8 instances concerning the hypervolume. According to the ε -add indicator the clean method achieved better results than the *crowdd* and random methods for 8 and 10 instances, respectively.

The statistical test showed that the variant with the random method performed worse than the other variants on instances KS100-3 and KS100-4. Significant differences between the variants were not indicated by the KW test for the remaining instances.

In average the clean, *crowdd*, random and *verif* variants spent 0.90, 4.05, 4.05 and 4.04 seconds, respectively.

6.2.2 mQAP

Table 7 shows the average results of hypervolume and ε -add achieved by the NSGA-II variants for the mQAP.

The *verif* version presented the best results for 25 from the 36 instances tested regarding both quality indicators. The best performance of *verif* can be observed for the 3 and 4-objective instances. The best results for the 12 2-objective instances, according to the hypervolume indicator, came from the random and clean methods. Differences due to the ε -add indicator were small for the 2-objective instances. The statistical test did not point out significant differences between the methods for the 2-objective instances.

Verif presented the best hypervolume and ε -add results for all 3 and 4-objective instances. Statistical differences between clean and *verif* were pointed out by the Kruskal-Wallis test for 21 instances. Statistical differences were also pointed out for *verif* and *crowdd* for all 3 and 4-objective instances. For 3 objective instances, the behaviors of *verif* and random were statistically different for 9 instances. Statistical differences between *verif* and random were not verified for 4-objective instances. A notable point concerns the instance size: the bigger the instance, the better is the performance of the *verif* and random methods.

Table 6. Results of the NSGA-II variants for the mKP instances

Inst	Hypervolume					$\varepsilon - add$			
	factor	clean	crowdd	random	verif	clean	crowdd	random	verif
KS100-2	10^5	1.461	1.495	1.426	1.464	123.6	112.5	128.2	121.0
KS100-3	10^8	2.386	2.298	2.189	2.482	216.0	221.0	266.6	213.7
KS100-4	10^{11}	1.931	1.535	1.865	2.066	265.8	307.2	290.1	249.6
KS250-2	10^8	2.545	2.344	2.209	2.224	406.5	408.4	430.4	450.7
KS250-3	10^8	4.687	4.587	4.884	5.064	457.8	433.6	425.1	437.2
KS250-4	10^{11}	7.074	6.005	6.159	7.284	497.7	539.4	552.6	515.5
KS500-2	10^5	1.635	2.033	2.248	2.238	647.4	571.8	576.4	544.3
KS500-3	10^8	1.662	1.888	1.174	1.461	878.8	821.5	900.7	841.1
KS500-4	10^{11}	7.351	7.608	8.126	8.414	807.5	809.6	822.3	820.0
KS750-2	10^4	9.391	6.316	6.089	4.086	926.7	1011.5	1050.1	1001.7
KS750-3	10^7	6.466	4.519	5.804	4.587	1002.4	1099.4	1063.9	1030.0
KS750-4	10^{12}	1.471	0.964	1.571	2.054	1070.7	1180.3	1088.0	980.6

Table 7. Results of the NSGA-II variants for the mQAP instances

Inst	Hypervolume					$\varepsilon - add$			
	factor	clean	crowdd	random	verif	clean	crowdd	random	verif
mqa1021	10^9	1.265	1.243	1.226	1.234	6204.5	6071.7	6814.7	6123.3
mqa1022	10^9	2.501	2.422	2.431	2.460	4597.6	5672.5	5520.5	5309.3
mqa1023	10^9	1.678	1.682	1.682	1.668	6286.3	6265.5	5913.8	6116.1
mqa2021	10^{10}	1.154	1.142	1.168	1.172	33876.7	33572.3	33267.5	32931.9
mqa2022	10^9	4.806	4.594	4.540	4.650	28539.5	29170.9	30489.5	31014.5
mqa2023	10^{10}	1.195	1.129	1.203	1.189	30830.0	32658.5	28803.3	29777.5
mqa3021	10^{10}	2.171	2.141	2.122	2.126	50318.6	54437.5	53842.9	51610.5
mqa3022	10^{10}	3.272	3.274	3.328	3.292	75914.6	70599.4	72956.4	72440.7
mqa3023	10^{10}	3.251	3.247	3.236	3.200	55244.6	50616.3	52529.9	55413.1
mqa4021	10^{10}	6.167	6.092	6.449	6.394	87069.6	88120.1	85054.9	85965.6
mqa4022	10^{10}	2.589	2.433	2.669	2.626	65178.8	69125.1	69172.5	69012.5
mqa4023	10^{10}	2.363	2.115	2.464	2.361	74865.3	74701.3	75106.5	68633.5
mqa1031	10^{14}	1.112	1.017	1.131	1.207	11496.4	15102.0	11046.8	9048.5
mqa1032	10^{14}	1.255	1.080	1.310	1.399	11949.6	17103.2	11065.3	9142.2
mqa1033	10^{13}	7.118	6.153	7.210	7.741	9970.8	13920.1	10381.0	8722.8
mqa2031	10^{15}	2.757	2.505	2.951	3.016	49317.8	59213.5	50501.9	47373.2
mqa2032	10^{15}	2.441	2.410	2.496	2.716	48570.4	53621.7	48060.7	42354.7
mqa2033	10^{15}	1.928	1.658	1.999	2.261	47941.0	58536.1	49637.3	41672.8
mqa3031	10^{16}	1.214	1.076	1.291	1.333	86366.4	104042.0	81343.5	77931.8
mqa3032	10^{16}	1.086	0.981	1.108	1.277	86214.9	100114.0	81861.5	73268.8
mqa3033	10^{16}	1.458	1.296	1.519	1.622	91467.9	105700.0	85098.5	80868.8
mqa4031	10^{16}	3.012	2.650	3.362	3.559	130372.0	148274.0	117812.0	110192.0
mqa4032	10^{16}	2.382	2.160	2.703	2.927	133825.0	146455.0	124835.0	117630.0
mqa4033	10^{16}	4.324	4.190	4.821	5.123	146348.0	154876.0	130113.0	119700.0
mqa1041	10^{18}	4.627	3.274	5.125	5.795	17486.8	23927.9	15413.7	13362.6
mqa1042	10^{18}	6.470	4.447	7.964	8.803	19977.9	27342.9	16622.7	14682.1
mqa1043	10^{18}	5.718	3.826	7.174	7.948	19052.5	26815.8	15452.4	13682.0
mqa2041	10^{20}	4.346	3.869	5.272	5.715	72735.9	84229.3	62832.5	58426.7
mqa2042	10^{20}	3.722	3.171	4.907	5.454	73489.3	82557.4	61448.9	57157.9
mqa2043	10^{20}	6.145	5.482	7.318	8.313	68401.9	78180.9	61099.3	57071.7
mqa3041	10^{21}	3.732	3.371	4.272	5.204	124921.0	133750.0	111912.0	100329.0
mqa3042	10^{21}	2.689	2.480	3.557	3.958	128565.0	142696.0	112369.0	108146.0
mqa3043	10^{21}	4.151	3.618	4.460	5.106	120551.0	134303.0	113570.0	105361.0
mqa4041	10^{22}	1.982	1.755	2.268	2.553	188274.0	202735.0	169359.0	161579.0
mqa4042	10^{22}	1.656	1.550	2.012	2.161	184825.0	196702.0	159827.0	157626.0
mqa4043	10^{22}	1.974	1.844	2.404	2.584	190013.0	204006.0	160776.0	163512.0

Figure 1 shows the hypervolumes calculated for 2 instances: mQAP4021, which has 40 facilities and 2 objectives, and mQAP4041, which has 40 facilities and 4 objectives. This figure illustrates the general behavior of the algorithm for 2 and 4-dimensional instances. For the 2-dimensional instance, the figure shows that the hypervolume curves of the

4 algorithmic variants are close one another. However, for 4-dimensional instances, the random and verif methods stood up along the optimization.

In average the clean, crowdd, random and verif variants spent 0.61, 1.92, 2.06 and 2.08 seconds, respectively. When the number of objectives was 2 or 3, crowdd method was faster

Figure 1. Hypervolume evolution calculated for the NSGA-II variants applied to the mQAP.

than random and verif, and spent from 0.2 to 0.4 seconds less.

6.2.3 Continuous problems

Table 8 shows the results of the NSGA-II variants for continuous instances. The DTLZ problems are named $DTLZ_N^d$, where d is the number of objectives. The hypervolume values presented for the DTLZ instance has to be multiplied by 10^{11} .

In general, the verif variant generated the best approximation sets, concerning the two quality indicators. The statistical tests for the hypervolume values indicated that the verif method outperformed the others on all 4-objective DTLZ instances and the DTLZ2³, DTLZ5², LFZ09F1, LFZ09F6, WFG8, WFG9, Fonseca, Kursawe and Viennet3 instances. The crowdd and random methods were outperformed by the verif and clean on the DTLZ, Fonseca, Kursawe and Viennet3 instances. The statistical tests for the ϵ -add values indicated that the verif method outperformed the other methods on the DTLZ6⁴ and LFZ09F1 problems and that, in general, the crowdd and random methods were outperformed by the verif and clean.

In summary, for the continuous problems, the best results were obtained by the verif variant, followed by the standard

NSGA-II algorithm. The clean variant did not outperform the verif variant on any problem regarding both quality indicators. The crowdd method presented the worst results, in general.

The average times spent were 0.58, 2.50, 2.54 and 2.60 seconds respectively by clean, crowdd, random and verif methods.

6.2.4 Discussion about the NSGA-II variants results

The crowdd method, based on the crowding distance brought no benefit to the NSGA-II. This fact showed that double applying the crowding distance to select nondominated solutions for the NSGA-II also magnified the deterioration effects. In general, the verif method improved the NSGA-II results regarding the discrete and continuous optimization problems concerning both quality indicators. The random method improved the NSGA-II results for the mQAP instances.

6.3 Recycling with the SPEA2

The SPEA2 (Strength Pareto Evolutionary Algorithm), proposed by [1], is also a multi-objective genetic algorithm. SPEA2 maintains a regular population and an external archive which stores the non-dominated solutions generated during the algorithm execution. The fitness of the solutions within the population and the external archive are computed every iteration. The fitness $F(i)$ of the i -th solution is the sum of two parcels: the ratio between the number of solutions dominated by the i -th solution and the population size, and an estimation based on the closeness among solutions. The aim of the latter parcel is to ensure diversity. The selection method of the SPEA2 is explained in Section 3.2.

6.3.1 mKP

The results of the SPEA2 variants applied to the mKP are presented in Table 9.

Table 9 shows that the 3 recycling variants present better results for both indicators than the standard SPEA2 algorithm. The crowdd variant presented the best results for 6 and 8 instances from the 12 tested, regarding the hypervolume and ϵ -add indicators, respectively. The statistical test pointed out significant differences for the 3 recycling variants in comparison with the clean variant for the KS100-2 and KS100-4 instances. The crowdd variant was also statistically different from the clean variant on the KS100-3, KS250-2 and KS250 instances.

The average processing times were 2.01, 4.83, 4.84 and 4.94 seconds, for clean, crowdd, random and verif methods.

The quality indicators computed for the approximation sets generated by the SPEA2 applied to the mQAP are shown in Table 10.

6.3.2 mQAP

For the hypervolume indicator, Table 10 shows that except the mqap2021 instance, the best results for all instances were found by the recycling techniques. The KW test indicated favorable results over the clean version by some recycling technique on 23 instances. The crowdd version presented significant better results than the clean for 5 instances concerning

Table 8. Results of the NSGA-II variants for the continuous instances

Problem	Hypervolume				$\varepsilon - add$			
	clean	crowdd	random	verif	clean	crowdd	random	verif
DTLZ1 ²	0.115	0.110	0.102	0.119	0.0230	0.0515	0.0641	0.0173
DTLZ1 ³	0.046	0.057	0.056	0.064	0.2763	0.2036	0.2290	0.2110
DTLZ1 ⁴	118845	112984	120365	120641	5.7120	6.2058	5.8470	4.3925
DTLZ2 ²	0.204	0.183	0.201	0.205	0.0227	0.0766	0.0833	0.0211
DTLZ2 ³	0.499	0.471	0.494	0.511	0.1635	0.1883	0.2608	0.1671
DTLZ2 ⁴	4.899	4.398	4.743	4.998	0.2695	0.5183	0.3487	0.2586
DTLZ3 ²	0.021	0.007	0.005	0.003	2.3116	2.4806	2.1886	2.3804
DTLZ3 ³	220.954	168.196	189.918	184.941	7.0378	7.6683	8.1164	7.7836
DTLZ3 ⁴	1.845	1.82	1.847	1.847	73.0515	83.0216	62.0410	56.3366
DTLZ4 ²	0.095	0.086	0.094	0.095	0.5451	0.5673	0.5626	0.5439
DTLZ4 ³	0.324	0.307	0.326	0.332	0.3009	0.3224	0.3683	0.3039
DTLZ4 ⁴	0.875	0.690	0.864	0.915	0.3361	0.4354	0.4139	0.3225
DTLZ5 ²	0.204	0.180	0.201	0.205	0.0227	0.0859	0.0858	0.0209
DTLZ5 ³	0.044	0.038	0.043	0.045	0.0206	0.0920	0.0696	0.0204
DTLZ5 ⁴	6.876	6.679	6.024	6.936	0.1659	0.2709	0.2061	0.1442
DTLZ6 ²	0.393	0.371	0.391	0.386	0.0729	0.1202	0.0950	0.0757
DTLZ6 ³	4.042	2.588	3.065	4.413	0.5235	0.8267	0.6631	0.4796
DTLZ6 ⁴	6212.710	5687.100	5878.070	6699.650	2.3126	3.2298	2.7051	1.9240
DTLZ7 ²	0.458	0.450	0.449	0.459	0.1872	0.2044	0.2047	0.1857
DTLZ7 ³	0.772	0.723	0.754	0.806	0.6607	0.6949	0.7302	0.5958
DTLZ7 ⁴	4.583	4.136	4.478	4.852	0.6024	0.7392	0.7684	0.5531
LZ09F1	0.762	0.750	0.760	0.770	0.0352	0.0760	0.0515	0.0287
LZ09F2	0.475	0.478	0.474	0.478	0.2737	0.2701	0.2832	0.2707
LZ09F3	0.619	0.607	0.601	0.624	0.1882	0.1925	0.2323	0.1846
LZ09F4	0.770	0.740	0.754	0.773	0.1570	0.1749	0.1792	0.1565
LZ09F5	0.628	0.610	0.613	0.633	0.1373	0.1393	0.1658	0.1333
LZ09F6	1.507	1.411	1.516	1.518	0.0427	0.1292	0.0130	0.0059
LZ09F7	0.746	0.745	0.738	0.745	0.2795	0.2818	0.2882	0.2796
LZ09F8	1.815	1.813	1.818	1.815	0.3690	0.3735	0.3675	0.3690
LZ09F9	0.158	0.154	0.156	0.158	0.3130	0.3156	0.3227	0.3152
WFG1	34.723	34.836	30.488	35.026	0.5531	0.8562	0.7101	0.5650
WFG2	41.515	38.793	42.395	42.682	0.4678	0.8297	0.3874	0.4143
WFG3	15.896	12.485	14.763	16.185	0.3192	0.8679	0.3925	0.2875
WFG4	15.876	15.278	16.009	17.140	0.5579	0.6757	1.1390	0.4942
WFG5	16.366	15.643	16.468	16.964	0.5360	0.6357	0.8300	0.5177
WFG6	15.271	14.271	15.220	16.028	0.5590	0.6560	1.1504	0.5437
WFG7	15.435	15.325	16.390	17.058	0.5778	0.6297	0.9769	0.5185
WFG8	10.375	8.959	10.715	11.770	0.7698	0.8397	1.6226	0.6987
WFG9	15.459	14.517	15.916	16.477	0.5874	0.7013	0.9333	0.5253
Fonseca	0.288	0.282	0.288	0.293	0.0261	0.0555	0.0481	0.0222
Kursawe	193.376	192.945	191.612	193.533	0.1642	0.3216	0.9525	0.1422
Viennet2	273.849	270.389	273.541	273.898	0.0537	0.2684	0.0309	0.0493
Viennet3	3.795	3.727	3.729	3.817	0.0737	0.1261	0.1381	0.0594

Table 9. Results of the SPEA2 variants for the mKP instances

Inst	Hypervolume					$\varepsilon - add$			
	factor	clean	crowdd	random	verif	clean	crowdd	random	verif
KS100-2	10 ⁵	0.955	1.198	1.102	1.119	200.0	128.8	155.2	148.3
KS100-3	10 ⁸	1.887	2.362	2.209	2.272	282.6	218.3	247.9	248.9
KS100-4	10 ¹⁰	5.612	7.778	7.995	8.237	297.9	226.3	242.9	233.7
KS250-2	10 ⁵	4.278	5.061	4.794	4.729	432.1	354.8	381.8	347.3
KS250-3	10 ⁸	1.392	1.608	1.504	1.338	410.8	365.8	385.2	403.2
KS250-4	10 ¹¹	1.306	1.791	1.603	1.679	554.1	441.6	486.6	460.7
KS500-2	10 ⁵	1.807	1.946	1.579	1.646	678.7	672.3	708.9	725.0
KS500-3	10 ⁸	1.602	1.556	1.620	1.583	818.1	717.3	803.8	762.4
KS500-4	10 ¹²	1.293	1.498	1.457	1.542	867.8	841.1	881.8	802.2
KS750-2	10 ⁴	2.865	2.605	6.096	3.073	1066.6	1026.6	978.6	1048.8
KS750-3	10 ⁸	2.880	3.254	3.361	3.581	1172.5	1079.3	1098.5	1022.3
KS750-4	10 ¹¹	4.992	5.582	5.675	5.398	1030.9	972.2	1017.1	1005.2

the two indicators and for one more instance concerning the ε -add. The random version also outperformed the clean version on 12 instances concerning the two indicators and another

instance according to the hypervolume. Finally, the verif versions presented better results than the clean version on 18 instances concerning both quality indicators and on other 3

Table 10. Results of the SPEA2 variants for the mQAP instances

Inst	Hypervolume					$\varepsilon - add$			
	factor	clean	crowdd	random	verif	clean	crowdd	random	verif
mqap1021	10^9	1.055	1.203	1.187	1.191	12731.7	6778.6	7822.2	8757.2
mqap1022	10^9	2.223	2.373	2.404	2.341	9477.8	6421.5	6427.8	8099.4
mqap1023	10^9	1.465	1.611	1.606	1.571	10436.9	6647.5	7593.6	8633.2
mqap2021	10^9	8.777	9.781	9.700	9.180	45727.5	35309.4	35960.7	40539.2
mqap2022	10^{10}	1.111	1.184	1.195	1.166	38045.7	30397.5	31488.1	33253.5
mqap2023	10^9	9.086	9.466	9.781	9.403	38005.5	32716.0	31860.1	35219.7
mqap3021	10^{10}	2.027	1.965	1.981	1.962	74739.5	70833.0	72360.9	80784.7
mqap3022	10^{10}	0.996	1.054	1.009	1.085	62192.5	57116.4	60214.8	60118.9
mqap3023	10^{10}	2.220	2.101	2.290	2.270	58264.9	59330.5	52420.7	57003.6
mqap4021	10^{10}	4.574	4.820	4.849	4.981	105791.0	87615.4	92224.7	93872.7
mqap4022	10^{10}	3.225	3.467	3.427	3.350	90981.0	80097.7	88521.1	89989.6
mqap4023	10^{10}	3.607	3.512	3.570	3.711	102051.0	104229.0	97381.0	99586.8
mqap1031	10^{14}	1.058	1.119	1.183	1.191	12847.3	12191.2	9835.3	9636.1
mqap1032	10^{14}	1.186	1.261	1.347	1.373	13340.4	13783.9	10429.7	10092.7
mqap1033	10^{13}	6.633	6.877	7.265	7.455	12324.9	11393.5	10440.6	10015.3
mqap2031	10^{15}	1.532	1.636	1.743	1.660	48522.8	44335.5	43491.7	46268.1
mqap2032	10^{15}	1.107	1.292	1.284	1.370	48882.9	43313.7	43473.1	41757.0
mqap2033	10^{15}	1.219	1.342	1.416	1.390	51228.5	41695.2	43473.8	43132.7
mqap3031	10^{16}	1.017	1.111	1.084	1.120	74276.7	68915.0	67925.1	67626.6
mqap3032	10^{15}	5.172	5.196	6.113	6.131	80058.1	76521.7	67754.0	65246.9
mqap3033	10^{15}	5.387	6.167	6.149	6.292	83869.3	69493.7	75876.2	75592.3
mqap4031	10^{16}	2.224	2.452	2.453	2.564	116142.0	104134.0	105021.0	107066.0
mqap4032	10^{16}	1.118	1.219	1.237	1.203	112239.0	96531.4	100374.0	112237.0
mqap4033	10^{16}	1.253	1.308	1.397	1.396	110470.0	100497.0	100212.0	104495.0
mqap1041	10^{18}	4.286	4.598	5.638	5.920	18181.0	18667.2	14001.2	13424.7
mqap1042	10^{18}	6.561	6.633	8.537	8.944	20229.7	21937.7	15211.5	15769.6
mqap1043	10^{18}	5.053	4.932	6.834	7.182	19564.9	21424.3	14702.4	13881.1
mqap2041	10^{20}	2.692	2.614	3.115	3.478	59898.3	64039.1	52576.8	50855.3
mqap2042	10^{20}	2.646	2.819	3.152	3.385	65731.6	59361.9	54420.3	57334.7
mqap2043	10^{20}	1.832	1.680	2.356	2.667	61901.1	64503.7	50726.0	50522.9
mqap3041	10^{21}	1.471	1.222	1.683	1.786	98014.9	108609.0	85961.4	85391.7
mqap3042	10^{21}	1.511	1.504	1.606	1.900	94906.7	100527.0	88206.8	83963.2
mqap3043	10^{21}	0.766	0.612	0.798	1.040	103001.0	107707.0	91535.1	90588.9
mqap4041	10^{21}	4.606	4.110	5.653	7.223	155401.0	168194.0	134847.0	129146.0
mqap4042	10^{21}	4.256	4.083	4.451	5.855	136026.0	143045.0	126956.0	122154.0
mqap4043	10^{21}	6.388	5.930	5.946	8.322	136890.0	148388.0	136348.0	120104.0

instances concerning the hypervolume. The KW test did not point out best significant results for the clean version on any instance.

The major contributions for the standard algorithm came from the crowdd method on small instances and the verif method on large instances.

On mQAP instances, the clean method spent, in average, 2.2s when $d = 2$ and $d = 4$, and 1.75s when $d = 3$. The average processing times of the recycling methods were 2.01, 4.04, 3.91 and 4.03, for clean, crowdd, random and verif, respectively.

6.3.3 Continuous Problems

Table 11 presents the results of the SPEA2 variants for the continuous problems. DTLZ1⁴ hypervolumes have to be multiplied by 10^6 , DTLZ3³ by 10^5 and DTLZ3⁴ by 10^{11} .

Concerning the hypervolume indicator, Table 11 shows that a recycling technique produced the best results. Significant statistical differences between the verif and clean methods were pointed out by the KW test for 16 from 21 DTLZ instances, 3 from 9 LZ09 instances, 8 from 9 WFG instances and for the Fonseca, Kursawe and Viennet3 instances. Signif-

icant differences were found for 4 DTLZ instances, 1 LZ09 instance and 2 WFG instances between clean and random, and for 2 LF09 instances between crowdd and clean.

Concerning the ε -add indicator, the recycling techniques presented the best results for 17 DTLZ instances. Significant differences were pointed out in favor of the verif method compared with the clean method for 7 DTLZ instances, 1 LZ09 instance, 3 WFG instances and for the Fonseca, Kursawe, Viennet2 and Viennet3 instances. Significantly favorable results for the crowdd method compared with the clean method were observed for 3 LZ09 instances and 4 WFG instances. The random method was statistically better than the clean method for 1 LZ09 instance and 1 WFG instance.

The average processing times of the recycling methods were 1.70, 4.00, 3.99 and 4.08 seconds for clean, crowdd, random and verif respectively.

6.3.4 Discussion about the SPEA2 variants results

The proposed recycling techniques significantly improved the SPEA2 results. Different from the NSGA-II, the crowdd method improved the outcome of the optimizer. We can note that crowdd method outperformed the other methods for half

Table 11. Results of the SPEA2 variants for the continuous problems

Problem	Hypervolume				$\varepsilon - add$			
	clean	crowdd	random	verif	clean	crowdd	random	verif
DTLZ1 ²	0.099	0.092	0.109	0.105	0.0861	0.1174	0.0528	0.0666
DTLZ1 ³	58.952	58.685	59.245	59.247	0.1627	0.5957	0.1395	0.1383
DTLZ1 ⁴	2.976	2.823	3.010	3.010	9.1186	19.9041	3.1314	2.4190
DTLZ2 ²	0.203	0.192	0.210	0.208	0.0618	0.0787	0.1046	0.0498
DTLZ2 ³	0.948	0.918	0.952	0.985	0.2162	0.2303	0.2452	0.2074
DTLZ2 ⁴	3.432	3.178	3.465	3.577	0.3661	0.4882	0.3773	0.3426
DTLZ3 ²	0.093	0.029	0.228	0.077	3.1664	3.9021	3.3059	3.3409
DTLZ3 ³	1.212	1.214	1.218	1.217	7.2194	6.4928	6.1686	5.9910
DTLZ3 ⁴	3.66164	3.61221	3.6666	3.6667	69.7490	138.0700	41.0422	35.9283
DTLZ4 ²	0.117	0.111	0.121	0.120	0.4287	0.4438	0.4601	0.4333
DTLZ4 ³	0.332	0.305	0.338	0.343	0.5943	0.6106	0.5923	0.5707
DTLZ4 ⁴	0.623	0.399	0.642	0.665	0.6043	0.7242	0.6054	0.6054
DTLZ5 ²	0.205	0.190	0.212	0.209	0.0618	0.0853	0.0869	0.0584
DTLZ5 ³	0.042	0.036	0.043	0.043	0.0492	0.1379	0.0750	0.0455
DTLZ5 ⁴	2.122	2.810	1.532	2.967	0.4450	0.3528	0.5567	0.2208
DTLZ6 ²	0.536	0.538	0.554	0.553	0.1575	0.1582	0.1374	0.1243
DTLZ6 ³	38.137	34.246	38.476	40.122	0.5470	0.9272	0.5263	0.3823
DTLZ6 ⁴	4130.730	3276.050	4189.770	4569.720	1.9922	3.8643	2.0774	1.4956
DTLZ7 ²	0.503	0.512	0.516	0.516	0.0678	0.0518	0.0488	0.0600
DTLZ7 ³	1.256	1.186	1.268	1.332	0.5455	0.5001	0.5352	0.4782
DTLZ7 ⁴	8.456	8.349	8.849	9.082	1.3501	1.0824	1.2966	1.1894
LZ09F1	0.632	0.640	0.670	0.674	0.0987	0.1165	0.0794	0.0634
LZ09F2	0.475	0.482	0.484	0.478	0.2719	0.2545	0.2569	0.2658
LZ09F3	0.659	0.684	0.669	0.686	0.2035	0.1572	0.2092	0.1829
LZ09F4	0.572	0.573	0.585	0.589	0.1049	0.1131	0.0977	0.0936
LZ09F5	0.675	0.694	0.681	0.696	0.1776	0.1313	0.1939	0.1598
LZ09F6	0.003	0.003	0.003	0.003	0.0018	0.0111	0.0000	0.0000
LZ09F7	1.551	1.624	1.606	1.618	0.4130	0.3758	0.3779	0.3768
LZ09F8	3.360	3.396	3.400	3.372	0.3665	0.3471	0.3505	0.3526
LZ09F9	0.159	0.156	0.157	0.162	0.3393	0.2865	0.3154	0.3043
WFG1	8.439	8.790	9.778	10.155	0.8048	0.8501	0.7673	0.6838
WFG2	39.378	33.672	41.186	41.334	0.5813	1.1026	0.3790	0.4481
WFG3	15.100	13.584	13.664	16.486	0.4802	0.8683	0.7779	0.3827
WFG4	14.412	15.258	15.920	16.341	0.8731	0.6735	0.9368	0.8216
WFG5	15.512	15.311	16.447	16.483	0.8934	0.6152	0.9068	0.7804
WFG6	14.924	15.115	15.971	16.509	0.9301	0.6055	1.1220	0.7971
WFG7	14.285	14.816	16.115	16.332	0.8297	0.6525	0.9764	0.7684
WFG8	11.384	10.737	13.651	14.180	1.1224	0.8909	1.3733	0.8754
WFG9	15.305	14.475	16.584	16.669	0.8075	0.6836	0.9939	0.7249
Fonseca	0.265	0.281	0.288	0.289	0.0945	0.0545	0.0570	0.0648
Kursawe	183.549	191.301	190.430	188.437	0.7915	0.6499	0.9770	0.6302
Viennet2	272.689	269.572	272.406	273.075	0.0462	0.2678	0.0307	0.0321
Viennet3	3.442	3.592	3.421	3.776	0.2944	0.2162	0.3065	0.1169

of the mKP instances concerning both quality indicators. Verif was the best method for the other problems. It obtained the best indicators values on most problems. The random method also improved the SPEA2 results for several instances, and in general, had better results than the clean SPEA2.

6.4 Recycling with PAES

PAES (Pareto Archived Evolution Strategy) is a multi-objective evolution strategy proposed by [2]. This algorithm is based on local search and maintains an external archive that stores nondominated solutions generated during its execution. The AGA technique is used to filter new solutions which are candidates to be inserted into the external archive. PAES has the three following main components. A solution generator which mutates a guide solution employing local search and generates a new candidate solution. A solution selector that determines which solution, among the guide and the new one,

is maintained for the next iteration. Finally, PAES maintains an external archive. As shown by [8], the external archive may deteriorate.

6.4.1 mKP

Table 12 shows the results of PAES $\rightarrow$ mKP. From the 12 mKP instances, the recycling techniques produced better hypervolume and ε -add values than the clean version for 10 instances. The statistical test pointed out a significant difference in favor of the random method compared with the clean version for instances KS100-4 and KS750-3.

The average times were 0.57, 0.61, 0.60, and 0.61 for the clean, random, crowdd and verif methods, respectively. The average processing times of the recycling methods were slightly greater than the clean method. This fact might be happened due to the solutions generated by the mutation process were not as good as the guide one, and the recycling bin stored few solutions.

Table 12. Results of the PAES variants for the mKP

Inst	Hypervolume					$\varepsilon - add$			
	factor	clean	crowdd	random	verif	clean	crowdd	random	verif
KS100-2	10^4	5.955	6.456	6.949	6.226	266.6	241.0	248.0	251.1
KS100-3	10^7	4.631	4.471	4.942	4.279	413.7	418.0	409.7	429.0
KS100-4	10^9	2.183	5.036	5.731	3.415	461.2	426.2	392.0	425.8
KS250-2	10^5	1.843	1.766	1.794	1.804	421.8	444.9	442.1	430.6
KS250-3	10^7	0.845	1.090	1.316	1.388	629.2	588.0	569.3	579.9
KS250-4	10^{10}	4.828	4.727	3.808	4.114	711.8	693.2	758.1	735.0
KS500-2	10^4	6.011	5.557	7.102	6.068	682.9	693.8	646.6	647.8
KS500-3	10^8	1.447	1.512	9.479	1.494	790.4	850.0	882.7	835.6
KS500-4	10^9	0.682	2.161	1.063	0.877	980.9	893.9	953.4	881.0
KS750-2	10^4	4.478	7.510	5.263	2.885	988.5	1014.0	973.7	1119.7
KS750-3	10^8	1.547	1.802	3.706	2.286	1461.9	1389.9	1213.8	1349.2
KS750-4	10^{11}	6.118	5.489	5.445	6.781	1517.9	1533.6	1490.6	1442.4

Table 13. Results of the PAES variants for the mQAP

Inst	Hypervolume					$\varepsilon - add$			
	factor	clean	crowdd	random	verif	clean	crowdd	random	verif
mqap1021	10^8	3.712	3.997	4.168	4.351	19760.4	18919.0	18322.3	19337.4
mqap1022	10^9	1.475	1.436	1.517	1.522	23532.5	23215.3	22143.9	22420.5
mqap1023	10^8	6.717	6.875	7.015	6.668	16598.5	15882.9	15647.1	16757.4
mqap2021	10^9	1.962	1.659	1.862	1.819	39391.2	42730.6	41038.1	41517.4
mqap2022	10^8	6.284	5.353	5.587	2.838	36812.7	38162.1	36073.0	40879.1
mqap2023	10^9	0.992	0.794	1.078	0.810	49910.3	48299.1	47905.8	47281.3
mqap3021	10^9	7.088	7.827	8.892	7.260	74147.3	70077.1	66773.6	79268.7
mqap3022	10^9	6.697	6.295	6.675	6.243	69962.4	71006.8	73399.5	71774.5
mqap3023	10^9	1.470	2.448	2.185	3.369	68988.1	63734.2	67674.4	61112.6
mqap4021	10^{10}	1.053	1.174	1.199	1.238	101063.0	104857.0	96199.0	101456.0
mqap4022	10^{10}	1.472	1.419	1.410	1.475	102192.0	91813.0	100846.0	95384.4
mqap4023	10^{10}	1.801	1.729	1.678	1.838	125341.0	122797.0	114639.0	115037.0
mqap1031	10^{13}	2.727	2.703	2.809	2.588	20508.1	19926.5	18477.4	19914.4
mqap1032	10^{13}	4.476	3.984	4.244	4.168	18874.2	21172.0	19410.7	20463.0
mqap1033	10^{13}	1.433	1.569	1.472	1.463	20413.6	19743.6	21317.5	20035.0
mqap2031	10^{14}	3.823	4.972	3.871	4.492	59236.7	54208.1	58907.5	56134.0
mqap2032	10^{14}	5.754	5.744	5.216	5.084	61505.1	61398.8	64693.1	64012.6
mqap2033	10^{14}	4.395	4.767	4.064	4.590	55399.2	53976.7	58613.3	53376.9
mqap3031	10^{15}	1.694	1.862	1.889	1.838	93889.8	91113.3	91770.8	97319.1
mqap3032	10^{15}	1.677	1.522	1.490	1.668	99701.2	99745.5	97415.5	97458.3
mqap3033	10^{14}	4.895	4.570	3.938	5.400	96976.6	97384.4	101892.0	90340.0
mqap4031	10^{15}	1.911	1.991	1.845	1.864	149312.0	140527.0	148034.0	143214.0
mqap4032	10^{15}	3.808	4.117	3.940	4.036	131867.0	121285.0	131277.0	123708.0
mqap4033	10^{15}	1.893	1.727	2.096	1.741	119054.0	122854.0	126375.0	130436.0
mqap1041	10^{17}	6.222	7.156	6.760	8.105	24245.1	24000.7	24245.8	23313.3
mqap1042	10^{18}	2.199	2.075	2.245	2.076	24114.9	25092.1	24736.9	24993.1
mqap1043	10^{18}	1.176	1.219	1.237	0.907	25709.3	27852.8	25633.6	27028.5
mqap2041	10^{19}	5.097	5.072	5.675	4.902	63237.5	64407.7	58227.1	65164.5
mqap2042	10^{19}	4.678	4.188	4.215	4.113	66012.0	66821.5	65626.4	69821.2
mqap2043	10^{19}	3.892	4.026	3.932	4.219	73037.1	71079.1	73457.2	72150.1
mqap3041	10^{20}	1.115	1.068	0.977	1.008	93269.0	92824.1	97945.0	92839.0
mqap3042	10^{20}	1.754	2.153	1.821	1.838	103954.0	87682.1	98771.0	97665.7
mqap3043	10^{20}	1.374	1.679	1.518	1.469	110804.0	100624.0	106420.0	107446.0
mqap4041	10^{20}	8.382	7.547	8.589	9.211	166838.0	161537.0	152980.0	155191.0
mqap4042	10^{20}	4.342	6.292	4.596	5.195	156363.0	138090.0	152010.0	153548.0
mqap4043	10^{20}	6.868	7.325	7.517	7.529	157845.0	144300.0	138431.0	157106.0

6.4.2 mQAP

Results of PAES $\rightarrow$ mQAP are presented in Table 13. From the 36 instances, the recycling techniques presented better hypervolume and ε -add values for 26 and 31 instances, respectively. Statistical differences were pointed out by the KW test for 3 instances: 2022, 2023 and 2031. The verif method was outperformed by the clean method on the 2022 instance. The crowdd method outperformed the clean method on the

2023 and 2031 instances. The average time results were 2.01, 1.98, 1.98 and 2.04 for the clean, random, crowdd and verif methods, respectively.

6.4.3 Continuous Problems

Table 14 shows the results obtained by PAES and its variants. WFG1 hypervolumes have to be multiplied by 10^{-5} .

No statistical difference was verified for the DTLZ1² and DTLZ2² instances. For $d = 3$ and $d = 4$, the crowdd method

Table 14. Results of the PAES variants for the continuous problems

Problem	Hypervolume				$\varepsilon - add$			
	clean	crowdd	random	verif	clean	crowdd	random	verif
DTLZ1 ²	0.045	0.043	0.042	0.045	0.1100	0.1106	0.1308	0.1058
DTLZ1 ³	0.038	0.049	0.042	0.039	0.2067	0.1988	0.2204	0.2078
DTLZ1 ⁴	0.702	0.803	0.706	0.701	0.1999	0.1884	0.2083	0.2001
DTLZ2 ²	0.105	0.088	0.089	0.105	0.1331	0.1356	0.2323	0.1315
DTLZ2 ³	0.054	0.079	0.063	0.053	0.3547	0.3448	0.3342	0.3564
DTLZ2 ⁴	0.027	0.044	0.034	0.026	0.3673	0.3392	0.3384	0.3706
DTLZ3 ²	0.016	0.016	0.016	0.016	1.0030	1.0034	1.0179	1.0030
DTLZ3 ³	0.023	0.023	0.022	0.022	0.9072	0.9019	0.9090	0.9494
DTLZ3 ⁴	0.436	0.443	0.410	0.426	0.8663	0.8169	0.8457	1.0512
DTLZ4 ²	0.0000	0.0000	0.0000	0.0000	1.027	1.027	1.027	1.027
DTLZ4 ³	0.000	0.000	0.000	0.000	0.8253	0.8253	0.8531	0.8194
DTLZ4 ⁴	0.000	0.000	0.000	0.000	0.9364	0.9364	0.9399	0.9425
DTLZ5 ²	0.107	0.090	0.095	0.107	0.1331	0.1356	0.2136	0.1352
DTLZ5 ³	0.022	0.015	0.020	0.023	0.1876	0.1506	0.2469	0.1678
DTLZ5 ⁴	0.031	0.030	0.024	0.031	0.2206	0.1891	0.2779	0.2283
DTLZ6 ²	0.089	0.076	0.073	0.089	0.1255	0.1285	0.2241	0.1255
DTLZ6 ³	0.215	0.211	0.151	0.215	0.1330	0.1231	0.2483	0.1290
DTLZ6 ⁴	0.325	0.327	0.203	0.325	0.4402	0.4060	0.5132	0.4373
DTLZ7 ²	0.254	0.200	0.245	0.254	0.1093	0.1254	0.1725	0.1060
DTLZ7 ³	0.231	0.229	0.202	0.234	1.3656	1.1149	1.1828	1.3521
DTLZ7 ⁴	0.374	0.414	0.316	0.396	2.6058	2.0166	2.1985	2.5543
LZ09F1	0.410	0.397	0.381	0.410	0.3339	0.3333	0.3730	0.3361
LZ09F2	0.031	0.031	0.031	0.031	0.3538	0.3538	0.3604	0.3529
LZ09F3	0.097	0.097	0.093	0.097	0.2715	0.2715	0.2893	0.2752
LZ09F4	0.024	0.024	0.024	0.024	0.3129	0.3125	0.3280	0.3092
LZ09F5	0.066	0.066	0.066	0.066	0.2718	0.2717	0.2806	0.2656
LZ09F6	0.004	0.004	0.004	0.004	0.5035	0.4812	0.5077	0.4929
LZ09F7	0.016	0.016	0.016	0.016	0.3062	0.3062	0.3062	0.3191
LZ09F8	0.415	0.415	0.415	0.415	0.3583	0.3583	0.3583	0.3583
LZ09F9	0.126	0.126	0.126	0.126	0.3858	0.3857	0.3858	0.3860
WFG1	5.750	5.886	4.409	6.291	0.4180	0.4163	0.4759	0.4121
WFG2	10.213	12.226	10.101	10.338	2.1665	1.1385	1.8536	2.1372
WFG3	4.578	4.098	3.895	4.586	0.7232	0.6466	0.7726	0.7341
WFG4	5.796	11.726	8.025	6.030	4.5725	2.2579	3.4247	4.4687
WFG5	7.006	10.869	7.256	6.489	4.0115	2.0612	3.1523	4.2826
WFG6	5.405	7.149	4.925	5.492	2.7511	2.1947	2.4678	2.6503
WFG7	5.818	7.479	5.068	5.740	3.2204	2.9108	3.3369	3.2291
WFG8	9.465	11.301	9.144	9.932	2.4381	2.2345	2.4002	2.3688
WFG9	4.911	7.677	6.321	4.750	4.1911	3.0298	3.9466	4.4855
Fonseca	0.268	0.238	0.202	0.270	0.1416	0.1808	0.4446	0.1505
Kursawe	185.293	185.375	177.479	186.269	1.2915	1.3283	2.6610	1.3345
Viennet2	275.147	272.548	267.882	275.257	0.0317	0.2355	0.0758	0.0306
Viennet3	2.929	2.367	2.931	2.878	0.4331	0.7484	0.8568	0.4536

outperformed the others concerning the hypervolume indicator. The statistical test did not show significant differences for the LZ09 functions. The crowdd method outperformed the other methods concerning both quality indicators on the WFG2, WFG4, WFG5 and WFG6 instances. It also outperformed the other methods concerning the hypervolume indicator on WFG7 and WFG9 instances. The clean and verif methods were the best ones for the Fonseca, Kursawe, Viennet2 and Viennet3 problems. The average processing times of the clean, crowdd, random and verif methods were 1.66s, 1.87s, 1.85s and 1.94s, respectively.

6.4.4 Discussion about the results of the PAES variants

The recycling techniques were less efficient for PAES than for the other optimizers. This fact may have occurred due to the nature of the algorithm which maintains a single solution in its population. The recycling methods only interfered on the

PAES archive, but the guide solution was not changed. For the previous algorithms, the recycling techniques interfered on the population.

7. Conclusions and Future Works

This study investigated methods to recycle nondominated solutions discarded by multi-objective optimizers. This investigation was motivated by the conclusions of previous studies presented by [8] and [9] which concluded that most filtering techniques used by multi-objective algorithms might deteriorate (or present other significant problems). Thus, this paper presented methods to prevent deterioration and improve the quality of the approximation sets generated by multi-objective optimizers. To accomplish this task, an archive of discarded solutions (recycling bin) was included to each optimizer. Three distinct techniques were proposed to man-

age the solutions of the recycling bin. The techniques were called *random*, *crowdd* and *verif*. Periodically solutions of the recycling bin were scanned to be inserted back to the optimizer. The recycling techniques were implemented into three known multi-objective algorithms: NSGA-II, SPEA2 and PAES. Computational experiments with 91 instances of discrete and continuous problems were done. The results showed that the capacity of the recycling techniques to lessen deterioration effects depended on the optimizer and the problem.

Based on the results of the experiments, the SPEA2 was the most benefited algorithm by the recycling techniques. The clean version performed poorly in comparison to the recycling variants. The least benefited algorithm was the PAES. It occurred since the recycling techniques did not directly interfere on the guiding solution. The recycling techniques only acted on the external archive. The best recycling technique for the NSGA-II was the periodical verification of deterioration (*verif* method). The recycling technique based on crowding distance (*crowdd* method) brought the best benefit for the PAES. A noteworthy fact is that the crowding distance did not present good results on the experiments for fixed input sequences, especially when those sequences were taken from problems with more than two objectives.

Future works will investigate other low complexity metrics to guide the recycling methods, such as those proposed by [30]. As at least one recycling method was able to improve the optimizers, future works will also investigate hybrid recycling methods and extend this study to many-objective problems.

Author contributions

All authors discussed the methods of recycling solutions on multi-objective optimizers and planned the experiments. H. G. de Medeiros implemented the algorithms and ran the experiments. E. F. G. Goldberg and M. C. Goldberg supervised the whole work. All authors analyzed the results and contributed to the final manuscript.

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