THE CONCEPTIONS AND CONTRIBUTIONS OF PESTALOZZI, GRUBE, PARKER AND DEWEY FOR TEACHING ARITHMETIC IN ELEMENTARY LEVEL: THE CONCEPT OF NUMBER

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Abstract

This article discusses the concepts related to teaching Arithmetic of Pestalozzi, Grube, Parker and Dewey. These concepts permeate time and leave traces in the textbooks used. Focusing research on the concept of number present in textbooks reveals important contributions of these authors in the constitution of Arithmetic as school subject in Brazil.

Key-words: Arithmetic, number, textbook.

AS CONCEPÇÕES E CONTRIBUIÇÕES DE PESTALOZZI, GRUBE, PARKER E DEWEY PARA O ENSINO DA ARITMÉTICA NO NÍVEL ELEMENTAR: O CONCEITO DE NÚMERO

Resumo

Este artigo aborda as concepções relacionadas ao ensino de Aritmética de Pestalozzi, Grube, Parker e Dewey. Tais concepções perpassaram os tempos e deixaram vestígios nos livros didáticos utilizados. Focalizar a investigação no conceito de número presente nos livros didáticos revela importantes contribuições destes autores na constituição da Aritmética como disciplina escolar no Brasil.

Palavras-chave: Aritmética, número, livro didático.

LAS CONCEPCIONES Y CONTRIBUCIONES DE PESTALOZZI, GRUBE, PARKER Y DEWEY PARA LA ENSEÑANZA DE LA ARITMÉTICA EN EL NIVEL ELEMENTAL: EL CONCEPTO DE NÚMERO

Resumen

Este artículo aborda los conceptos relacionados con la enseñanza de la Aritmética de Pestalozzi, Grube, Parker y Dewey. Estos conceptos impregnan tiempo y dejan huellas en los libros de texto utilizados. Investigación que se centra en el concepto de número presente en los libros de texto revela importantes contribuciones de estos autores en la constitución de la Aritmética como asignatura escolar en Brasil.

Palabras-clave: Aritmética, número, libro de texto.

LES CONCEPTIONS ET CONTRIBUTIONS DE PESTALOZZI, GRUBE, PARKER ET DEWEY POUR L'ENSEIGNEMENT DE L'ARITHMÉTIQUE DANS LE NIVEAU ÉLÉMENTAIRE: LE CONCEPT DE NOMBRE

Résumé
Cet article présente les concepts liés à l'enseignement de l'Arithmétique de Pestalozzi, Grube, Parker et Dewey. Ces concepts imprègnent temps et laissent des traces dans les manuels scolaires utilisés. Recherche axée sur le concept de nombre présent dans les manuels scolaires révèle d'importantes contributions de ces auteurs dans la constitution de l'Arithmétique comme une discipline scolaire au Brésil.
Mots-clé: Arithmétique, le nombre, des manuels scolaires.

Introduction

This article discusses the ideas of Pestalozzi, Grube, Parker and Dewey on the teaching of Arithmetic presented in the elementary level textbooks, from the 19th century. This study was conducted based on the research of textbooks used as documentary corpus of doctoral thesis entitled *A aritmética escolar no ensino primário brasileiro: 1890-1946*[1] (Costa, 2010). Most textbooks that were used in this study were obtained from the Bibliothèque Nationale de France, particularly the ones from 19th century.

Using instrumental theoretical-methodological founded the historic cultural studies, in particular the field of history of school subjects inaugurated by André Chervel as well as Alain Choppin's research on the history of textbook, this research was based on important original documents obtained during the period of research at the Institut National de Recherche Pédagogique - Service d'Histoire de l'Éducation. This allowed us to infer important contributions on the history of teaching of arithmetics in Brazil in the late 19th century and early 20th century.

This article discusses aspects of the constitution of the contents studied in school arithmetic, the influence of Pestalozzi in elementary school arithmetic, contributions of Grube and Parker, and finally, considerations of Dewey about the psychology of number.

The school arithmetic

One of the chapters of the book *Historia ilustrada del libro escolar en España*, organized by Agustín Escolano, the researchers Sierra Modesto Vazquez, Luis Rico Bernardo Romero and Alfonso Gómez have written the text *El numero y la forma: libros e impresos para la enseñanza del calculo y la geometria*. According to these authors, from the classical division of the disciplines of the *trivium* and *quadrivium* corresponding to the mathematics, Arithmetic emerged as one of its parts. Since the beginning of the Renaissance, concomitantly with the appearance of the press, a major expansion and dissemination of knowledge favored integration of mathematical thinking at that time in the uses and economic and social customs. The practical arithmetic was replaced the classical notion of logistics and, as a result, we started to refer to the other part as theoretical arithmetic:

> A partir del siglo 19 esta diferenciación entre las aritméticas teórica y práctica desaparecerá, unificándose ambas bajo el nombre común de aritmética, reservándose las coletillas de teórica y práctica para lo que es el planteamiento teórico de conceptos y propiedades, junto con la fundamentación, cuando es posible, de algunos procedimientos en el primer caso, y lo que es la aplicación o ejecución de los procedimientos y las reglas prácticas, en el segundo. (Vazquez et al., 1997, p. 374)

As a legacy of the Arabs, the calculation methods derived from the decimal numbering system are essentially configured as we know them today and the use of the

[1] The thesis was presented at the Postgraduate Studies Program in Mathematics Education from the Pontifical Catholic University of São Paulo, advisored by Prof. Dr. Wagner Rodrigues Valente and co-advisored by prof. dr. Alan Choppin during period of PhD research in Paris, France, in INRP/SHE.

figures had consequences, as highlighted by David Smith in *The teaching of elementary mathematics*:

> Arithmetic, at least in the Western world, was always based upon object teaching until about 1500, when the Hindu numerals came into general use. But in the enthusiasm of the first use of these symbols, the Christian schools threw away their abacus and their numerical counters, and launched out into the use of Hindu figures. And while they saw that the old-style objective work was unnecessary for calculation, which is true, they did not see that it was essential as a basis for the comprehension of number and for the development of the elementary tables of operation. Hence it came to pass that a praiseworthy revolution in arithmetic brought with a blameworthy method of teaching. (Smith, 1902, p. 71)

Vazquez et al (1997), bringing this discussion to the scope of textbooks in Arithmetic, claims that they show significant differences in the period that reaches the end of the 18th century, just as the form of presentation of knowledge, besides the greater or lower completeness that distinguished the character of manual or compendium, commercial or scholarly book. The systematic presentations of the various ways of calculating about the same transaction were the predominant, illustrating them with examples. The absence of arguments, it is understood today as the reasoning, was common in the books of this period. During the 18th century, in Europe, the books devoted to the Arithmetic used to incorporate the teaching of writing, grammar and spelling. In the early 19th century an important change in the configuration of this Arithmetic's school is produced due to the influences received from the French Revolution. In fact, the Revolution situated the mathematics in an elevated position, which resulted in a broad social diffusion.

Establishing a mandatory standard curriculum to the students of the same educational level implied the necessity of producing the textbooks. These books were characterized by a spirit elemental, which was understood as exposure of more essential, namely, the basic elements of knowledge: compendium, put in better order: method, more simply: soon, and so clearer this knowledge to be teachable. This connotation of the term essential come to be changed later, going to be viewed as a text or an abbreviated text that summarizes a dense text shorter than another. What is certain is that supposed a selection and organization of knowledge, which is organized in a proposed educational for children in which they included the Arithmetic and settled teaching of the count and the first rules. Before the French National Convention, there was a call for a selection of key books for public education. One of the four selected texts highlights the Condorcet, the key book to understand what has been the school Arithmetic (Vazquez et al, 1997).

This book brings three important contributions:

> la primera es el deseo de poner de manifesto la lógica de las reglas de cálculo y el análisis de los motivos que la sustentan; la segunda es la inclusión en el texto de sugerencias para los profesores; la tercera es un programa dividido en lecciones, las cuales, según él mismo, encierran cada una lo que es posible presentar en una sola sesión y no conviene separar en trozos. (Ibid., p. 377)

The contents

> eran: 1) La numeración escrita y en cifras y los signos + e =. 2) El sistema de numeración oral. 3) El sistema de numeración escrito (el valor de la posición y el papel del cero). 4) Adición. 5) Sustracción. 6) Pruebas de la suma y de la resta. 7) Multiplicación, significado y productos de números de una cifra. 8) El algoritmo para multiplicar números de varias cifras. 9) División, significado y uso. División por una cifra. 10) El algoritmo de la división por números de varias cifras. 11) División inexacta. Representación del resto en forma de fracción. Forma de leer una fracción. 12) Pruebas de la multiplicación y división. (Ibid., p. 377)

According to Vasquez et al (1997), in the course of time, this program has become classic and consolidated the identity of Arithmetic with the count, with the four rules of operations and with some rudiments of fractions.

Another important factor that influenced the school Arithmetic is the recognition of the theory of faculty psychology, which believed that the mind is composed of several faculties as imagination, memory, perception and reasoning. These are roughly analogous to the muscles, and that as such, must be exercised through training. In this frame of thought, explains the mental discipline as an educational goal that is achieved through a keen intellectual work around those disciplines that are considered most suitable for training the mind. The Arithmetic calculations were seen as good exercises to strengthen and mature the mind. Students must do without the use of pencil and paper, a lot of mental calculation exercises. They have become exhaustive and marked that time (ibid., 1997).

Pestalozzi and his methods

At the end of the 18th century, under the influence of Rousseau, the focus of education has turn to the students. It was not just getting the indoctrination of students, but the development of their natural abilities through methods considered more in accord with nature. The conception of teaching began to experience a change that was reflected in interest by formulating new methods equally suitable to the nature of children. Certainly the emblematic figure of this search for new methods more suitable for children was Pestalozzi[2], who tried to understand and improve the popular education (Eby,1962).

In Buisson (1887), there is an important mention:

> Henri Pestalozzi tient le premier rang parmi ceux qui ont contribué à fonder la pédagogie moderne. Il est naturel que ce Dictionnarie consacre um article d'une certaine étendue à l'homme au nom duquel on rattache, à tort on à raison, presque tout ce qui a été fait dans le domaine de l'éducation depuis trois quarts de siècle (Buisson, 1887, I^{er}, t-2^{e}, p. 979).

In the origin of the renewal movements of teaching, Pestalozzi position is usually associated in the importance of his ideas about the education of children, intuition, using of learning objects in popular education. That is true, especially in regard to his ideas about the number and of the forms in math.

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[2] Johann Heinrich Pestalozzi (1746-1827) was born in Zurich. He has attracted the attention of the world as a teacher, principal and founder of schools. His major works were: *Leonardo e Gertrudes* (1781) and *Como Gertrudes instrui seus filhos* (1801). As a disciple of Rousseau, he was convinced of the innocence and the human kindness, and understood that the task of the teacher was to stimulate the spontaneous development of the student, seeking to understand the childlike spirit, which attitude away him from the dogmatic and authoritarian teaching.

Comenio and Rousseau, for example, have also contributed to the development of ideas about school education, however, their positions are expressed in distinct fields of Arithmetic. Pestalozzi undoubtedly has given an important place to the number and their learning. For him new ways of approaching the teaching of Arithmetic have been proposed and their students were getting amazing results in the learning of calculus (Gallego, 2005).

The Pestalozzi's proposed starts, like so many other reformers of education, the need to extend education to a large number of children, since the system of individual instruction used at the time was not appropriate to this proposed. Thus, Pestalozzi has recognized the need to implement a mutual educational system:

> Habiéndome visto obligado a instruir solo y sin auxilios a un gran número de niños, aprendí el arte de enseñar a los unos por medio de los otros, y como no tenía otro medio que la pronunciación en alta voz, concebí naturalmente el pensamiento de hacerlos dibujar, escribir y trabajar durante la clase. (Pestalozzi, 1980, p. 10)

But his primary concern was to discover a method that was effective and could be applied regardless of the competence of those who acted as a teacher:

> Esto es esencial. Yo creo que no hay que pensar en avanzar un paso, en general en la educación del pueblo, mientras no se hayan encontrado las formas de enseñanza que hacen del maestro, por lo menos hasta la conclusión de los estudios elementales, el simple instrumento mecánico de un método cuyos resultados deben nacer por la naturaleza de sus formas y no por la habilidad del que lo practica. (Pestalozzi, 1980, p. 30)

The most important contribution in this respect is the fact that such a method should be based on knowledge of the characteristics of children, especially the knowledge of psychological laws. Pestalozzi has tried to determine these laws through reflection on their teaching experience:

> Sentía que eran decisivas mis experiencias sobre la posibilidad de establecer la educación del pueblo sobre fundamentos psicológicos, de poner como base de ella conocimientos efectivos adquiridos por la intuición y desenmascarar la inanidad de ese lujo superficial de palabras de la enseñanza actual. (Ibid., p. 12)

Pestalozzi has tried to determine these essentials contents, which allowed organizing the education of the child following its progressive development in accordance with their nature. In their experiments, and in an effort to determine the source of knowledge, he has come to the conclusion that they were in the number, the form and the word and that these three issues would form the basic teaching aids that allow transforming confused notions in precise notions (Gallego, 2005).

On these three basic ways of teaching, Pestalozzi sought to the special procedures that could be applied, which constitute the reform applied in the schools then. Such innovations have caused strong impressions on those who visited their school (Gallego, 2005).

These impressions have become important factor of his fame at the time. Buisson (1887) describes a visit made to the Pestalozzi's establishment by a German merchant. The approval of this visitor is recognized in the text:

> Blochmann raconte dans ses Extraits de la Vie de Pestalozzi qu'un jour un riche négociant de Nuremberg vint visiter l'instituition du réformateur; il avait entendu vanter la facilité avec laquelle calculaient les élèves, et, pour s'en assurer, il demanda l'autorisation de leur poser um problème: c'était une règle de société très compliquée, à quatre proportions et où toutes les données étaient des fractions. Les enfants lui demandèrent si la question devait être résolue mentalement ou par écrit. "Mentalement, si vous l'osez", répondit-il étonné, et il prit lui-même du papier et de l'encre pour résoudre le problème. Il n'en avait pás encore fait la moitié que, de tous cotes, on criait:"J'ai trouvé!" Les réponses concordaient avec le résultat qu 'il obtint quelques instants après. Se tournant vers Pestalozzi, il lui dit alors: "J'ai trois garçons, je vous les enverrai aussitôt que je serai de retour chez moi. (Buisson, 1887, p. 316)

In preparing these special procedures, Pestalozzi sought to the development of education that would comply with "la marcha de la naturaleza en el desarrollo del género humano" (Pestalozzi, 1980, p. 110) and, therefore, produced a series of graded exercises based on intuition.

The importance of intuition is one of the greatest contributions of Pestalozzi: he considers it the foundation of all knowledge and the principle of instruction that must be respected by any form of education that you can use. This principle of intuition and how to carry it into practice has been a decisive factor in the changes over the last century what has been given to the teaching of mathematics and, in particular, in the teaching of Arithmetic (Gallego, 2005).

It must be noted that the exercises were not isolated prepared exercises, but they were "series graduadas y psicológicamente entrelazadas" (Pestalozzi, 1980, p. 17), taking into account the child's development. Pestalozzi and his collaborators have devoted efforts to the discovery of these series, applied to elementary notions of human knowledge and formed the basis of the application of his method and its extension (Gallego, 2005).

The number on the Pestalozzi's system

> El tercer medio elemental para obtener nuestros conocimientos es el número. Mas en tanto que el lenguaje y la forma emplean varios medios de instrucción subordinados a su circuito elemental para conducirnos a nociones claras y a la independencia intelectual que ellos tienen por objeto hacernos, el cálculo es el único medio de enseñanza que no comprende ningún medio subordinado; él aparece siempre, hasta el último límite de sus operaciones, como la consecuencia más sencilla de la facultad elemental que nos pone en estado de darnos cuenta cabal, en todas nuestras intuiciones, de las relaciones de cantidad, de las diferencias del más y del menos y de representarnos esas relaciones hasta el infinito con la precisión más clara. (Pestalozzi, 1980, p. 104)

Pestalozzi has considered the number as one of three basic ways for obtaining knowledge and, therefore, devoted special attention to their learning. As the word and the form need the number to be able to present themselves as clear intuitions, the number was considered as the only way that has no subordination:

> El sonido y la forma llevan a menudo y de diversas maneras en sí mismo el germen del error y de la ilusión. El número, nunca; sólo él conduce a resultados infalibles, […] alcanza con más seguridad el objeto de la instrucción, esto es, las nociones claras, debe ser considerado como el más importante. (Ibid., p. 104)

Taking into account the knowledge of the laws of development of the human spirit, Pestalozzi has claimed that it should practice it with great skill and high care:

> Ese medio de enseñanza se ha de poner generalmente en práctica y con un cuidado y una habilidad los más grandes, y que para alcanzar el último fin de la instrucción, es sumamente importante presentar este medio de enseñanza bajo formas por las cuales se pueden aprovechar todas las ventajas que pueden proporcionar en general a la enseñanza una psicología profunda y el conocimiento más vasto de las leyes del mecanismo del mundo físico. (Ibid., p. 105)

Pestalozzi has considered the intuition as the foundation of his method and organized the study of Arithmetic through the following list of teaching contents, conditioned by the principle of intuition:

> La enseñanza estaba condicionada por el principio de intuición: se estudiaban aquellas cuestiones aritméticas que ejemplificaban la importancia de este principio. Por tanto, se insiste en: el aprendizaje de los primeros números; las diversas relaciones entre los números hasta el 100, expresadas de forma verba; las relaciones entre fracciones expresadas de forma verbal; la aplicación de esas relaciones a la resolución, de forma verbal, de problemas; y, sin embargo, se encuentran poças indicaciones sobre el estúdio de la aritmética escrita. (Gallego, 2005, p. 67)

Arithmetic has its origin in the simple aggregation and subtraction of multiple units. “Uno y uno son dos, y uno de dos resta uno” (Pestalozzi, 1980, p. 105), this is considered by Pestalozzi the fundamental form of Arithmetic. The number is considered the abbreviation for an aggregation of units and the student will only have a clear understanding of each number when besides his name and his symbol, recognized as consisting of units that Pestalozzi shows organized in different ways.

The elementary teaching has presented the number as the sound of a word read or repeated by heart, a memorization, a numerical series or as a symbol. Pestalozzi has criticized this practice, considering it an apprenticeship of the empty words:

> Si, por ejemplo, aprendemos únicamente de memoria: tres y cuatro son siete, y en seguida contamos con ese siete como si supiésemos realmente que tres y cuatro son siete, nos engañamos a nosotros mismos, porque no tenemos ninguna idea de su verdad intrínseca, por cuanto no tenemos conciencia de su fondo material, el único que puede convertir para nosotros esa palabra vacía en una verdad palpable. (Pestalozzi, 1980, p. 105)

For him, learning the number for the child should begin by developing clear intuitions about it, in the sense intuitions about the quantity as a property of collections and the relations between the numbers derived from the composition and decomposition of the numbers.

This apprenticeship has begun at early ages with exercises that Pestalozzi claims, both in the *Livro de las madres* (1803), as in the *Cómo Gertrudis Enseña a sus hijos* (1801). These exercises were designed to be performed in plays in charge of the mothers:

> Yo comienzo por el Libro de las madres en mis esfuerzos por dar a los niños una impresión viva y durable de las relaciones de los números consideradas como variaciones reales y efectivas del aumento y de la disminución de la cantidad en los objetos que se encuentran a la vista de ellos. (Pestalozzi, 1980, p. 106)

The first suitable exercises for children were to ask for the amount of multiple collections: the body parts, the designed objects, the fingers, the pebbles, a few objects that have a hand. It was proposed to the children solve these issues by the count, who has learned by imitation, seeing an old people, particularly the mother, telling the distinct collections proposals.

Also, it was used the cutting boards syllabification as the collections and thus, it has joined it to learning the numbers to the words. The learning of the words was based on the numbers, because the child has asked for the amount of syllables in each word and the pronunciation of which has occupied the first place, the second place, etc. Other type of exercise appears here whose answer is an ordinal number, who was also aided by counting in this context.

The first collections were at least ten elements and they were held on different exercises that were intended to be the starter Arithmetic operations:

> Colocamos una tablita y preguntamos al niño: "¿Hay aquí muchas tablitas?" - El niño responde: "No, hay sólo una." En seguida agregamos una más y preguntamos: "Una y una ¿cuántas son?" - El niño responde: "Una y una son dos." Así se continúa, y se agrega al principio sólo una cada vez, después dos, tres, etc. (Ibid., p. 106)

Later, when students have understood the additions of the one and the one until to ten units and when they have learnt to express them easily, it has resumed up the questionnaire, but it has varied question:

> Cuando tú tienes dos tablitas, ¿cuántas veces tienes una tablita?"- El niño mira, cuenta y responde exactamente: "Cuando yo tengo dos tablitas, tengo dos veces una tablita. ¿Cuántas veces uno son dos?, ¿cuántas unos son tres?, etc. ¿Cuántas veces está contenido uno en dos?, ¿en tres? (Ibid., p. 107)

As soon as the student knew the simple and the elementary form of addition, multiplication and division, and had become familiar, through intuition, the nature of the forms of calculation, it sought to be done through another exercise, meet and become familiar to the original form of the subtraction:

> Se quita una de las diez tablitas que se han sumado y se pregunta: "Cuando de diez has quitado uno ¿cuántos quedan?"- El niño cuenta, encuentra nueve y responde: "Cuando de diez he quitado uno, quedan nueve". Se quita en seguida la segunda tablita y se pregunta: "Uno quitado de nueve, ¿cuántos son?" - El niño cuenta de nuevo, encuentra ocho y responde:"Uno quitado de nueve son ocho." Así se continúa hasta el fin. (Ibid., p. 107)

It has looked for preparing for the learning of Arithmetic operations with such activities. Thus, it has proposed up gradually increase or decrease a collection exercises, asking the child by remaining amount, preparation of addition or subtraction, or has considered themselves as the quantities divided into parts of an element and played up the situation by using the word *veces*, in preparation for multiplication and division.

In all cases, in order to solve these exercises, Pestalozzi has used only the simple perception and counting to which attaches great importance on the relationship between the numbers:

> Y cuando el niño se ha ejercitado tanto en contar con objetos materiales, y con los puntos y rayas que los sustituyen, cuando esas tablas fundadas puramente en la intuición lo permiten, el conocimiento de las relaciones reales de los números se robustece entonces tanto en su espíritu que las formas de abreviación por los números ordinarios, aun sin intuición, se hacen comprensibles para él de una manera increíble, porque sus facultades intelectuales están libres de confusión, de vacíos y de enigmas que resolver. De modo que, en el verdadero sentido de la palabra, se puede decir que el cálculo enseñado así es sólo un ejercicio de la razón y nunca un trabajo de la memoria, o un procedimiento mecánico y rutinero, pero que es un resultado de la intuición más clara y más exacta y no puede conducir sino a la adquisición de nociones claras. (Pestalozzi, 1980, p. 107)

The principle of the intuition is the theoretical principle that determines the way in which these exercises elaborating and justifying the techniques employed in its resolution: "il faut, avant de séparer de l'objet l'idée de son nombre, que l'enfant puisse voir ce nombre étroitement lié à l'objet" (Chavannes, 1809, p. 28). Therefore, all the exercises refer to the collections:

> Lorsque l'enfant aura été exercé à distinguer et à nommer ainsi un, deux, trois, les différens assemblages d'objets qu'on lui présente, il ne tardera pas à observer, que les mots un, deux, trois, demeurent toujours les mêmes ; tandis que, ceux de pierre, de noix, avec lesquels il les lie changent suivant qu'on lui montre les uns ou les autres de ces objets ; dès-là, il en viendra bientôt à séparer l'idée du nombre de celle de la chose, et, par là même, à s'élever à l'idée abstrait de la quantité, ou au sentiment net et précis du plus ou du moins, indépendant de la nature des objets qu'il a sous les yeux. (Chavannes[3], 1809, p. 28)

[3] The ideas of Pestalozzi were first spreaded in France, and later in Spain by Alex Chavannes. Chavannes, member of the Canton of Vaud, was an observer in the Berthoud House of Education where Pestalozzi has worked. Viewing the practice of his ideas as a method, a system, an ordered whole, he has published the first edition of his book named *Exposé de la méthode élémentaire de H. Pestalozzi,* in France, 1895. This book was translated into Spanish in 1807 by Don Eugenio de Luque, *Exposición del método elemental de Henrique Pestalozzi.* For this research a copy of the second French edition dated 1809 was used.

The exercises about the first ten numbers were included by Pestalozzi in the *Libro de las madres* and therefore they have corresponded to the teaching given to the children in their play. As the *Libro de las madres* was used in the Pestalozzi school, the exercises commented possibly should be the first which have performed their students to get their education.

The teaching of Arithmetic in Pestalozzi's system

The exercises about the first ten numbers of the *Libro de las madres* have prepared children to start the school learning of Arithmetic. From this time the students were related primarily to the abstract numbers and the induction principle formulated by Pestalozzi was manifested in the exercises taking place with reference collections of the scribbles with different organizations, according to the proposed exercise.

These collections, which have serve as a basis for intuitive integer operations, were organized as Pestalozzi called Tábua n. 1, on which have performed the exercises in Arithmetic. Two other tables were also devised based on the decomposition of a square into equal parts. These tables have formed the basis of the intuitive learning of fractions and the operations with them.

Learning Arithmetic with the Tábua n. 1

This table works with numbers until one hundred, giving the idea of the relationships that exist between them, in particular those that emerge of compositions and decompositions of collections, as well as multiple and divisor. The relationship between the numbers has considered each other as parts are represented by the use of fractions:

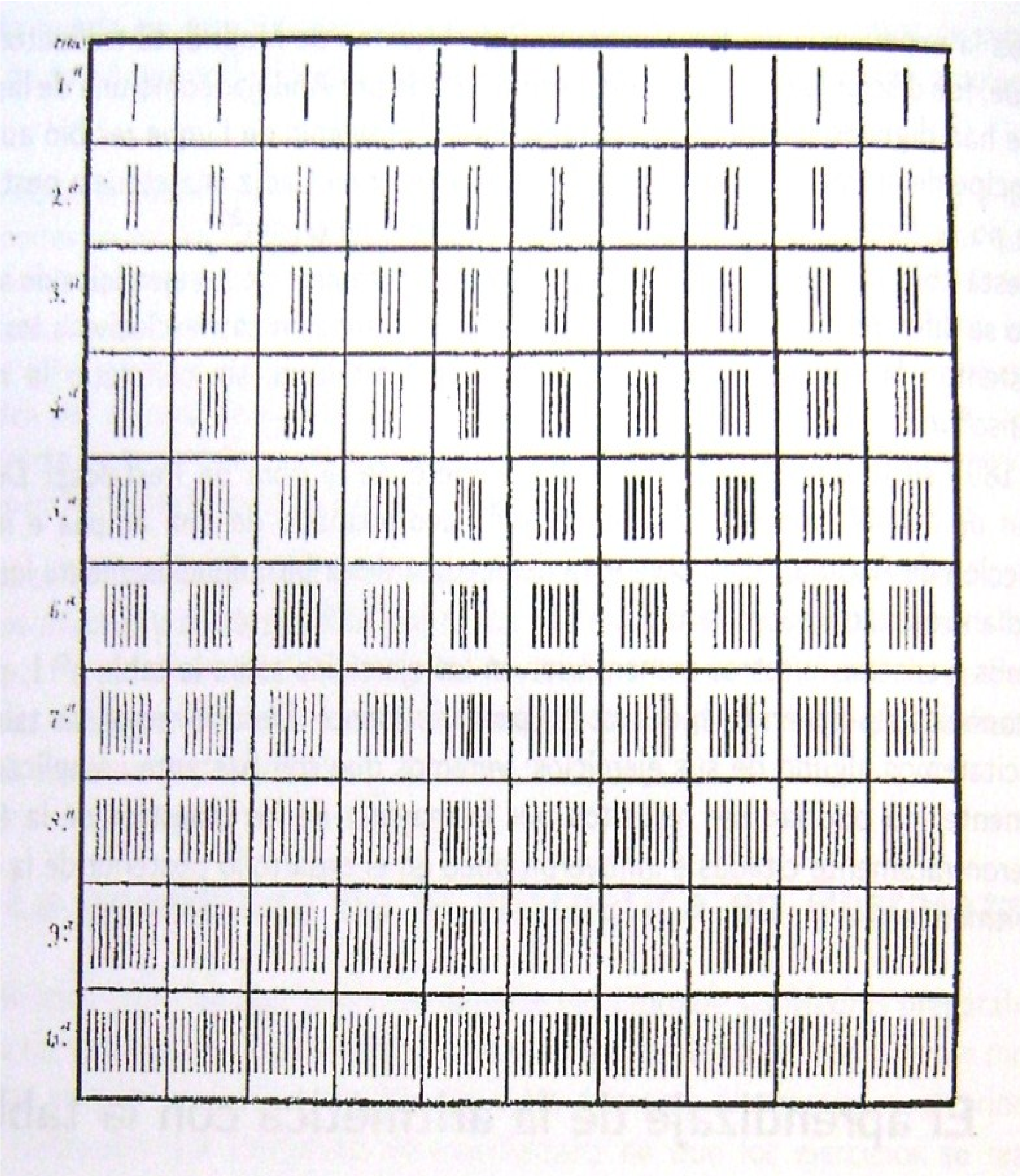
> Le but de ce premier tableau est d'exercer l'enfant, 1er.A voir l' unité, soit comme unité, soit comme faisant partie d'une somme d'unités. 2ème.A voir une somme d'unités, soit comme formant elle-même une unité, soit comme étant une partie d'une autre somme, et ainsi à comparer l'unité et chaque somme d'unités avec une autre somme, afin de préciser exactement leurs divers rapports. (Chavannes, 1809, p. 30)

The Tábua n. 1 is composed of collections of the scribbles arranged in ten rows. Each row consists of ten equal collections: the first row has a scribble; the second row has two scribbles and so on until the tenth row, which contains ten scribbles.

Eight different types of exercises that should be addressed in succession can be made by using this table. They form a graduation. In all of them is resorted to the table by rows, considering the different relationships between quantities that represent the scribbles.

The first exercise proposed, according to Chavannes (1809), is carried out in stages. At the start it requests that the student imitates the teacher, that performs the first exercise, and recognize that the first row is composed of collections of a scribble, the second row is composed of collections of two scribbles, on the third row of three and thus successively.

Figure 1
Pestalozzi's Tábua n. 1.



Source: Chavannes, 1809, p. 204.

After this exercise has done in a row, for example in the third, it asks them other exercise that consisting in reading the row as collection of three elements, for instance, once the three, two times the three, four times the three.

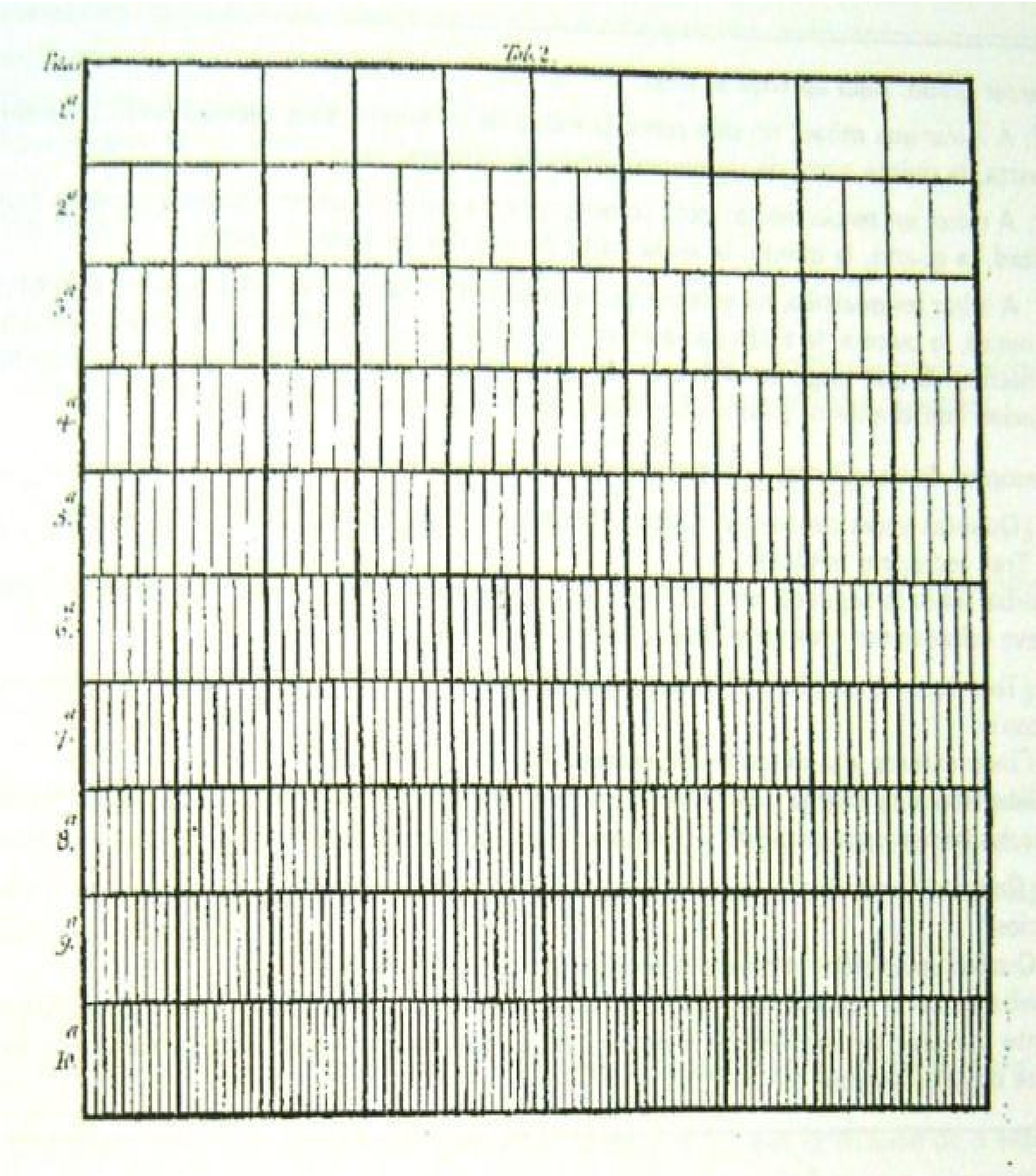
These exercises are performed by imitation of the teacher, aided by counting. The repetition of these exercises searches the memorization, so that the student is able to see how many two, three, four, etc., are on any one section of any row.

The other exercises that are presented in the text of Chavannes (1809) have higher degree of difficulty, but always they take the traits and houses as units.

Learning Arithmetic with the Tábua n. 2 and the Tábua n.3

The second table is composed of ten rows with ten equal squares each. The second row are divided into halves by a vertical line, with the third row into thirds, the fourth row into quarters and so on until the tenth row, which are divided into tenths always by vertical lines.

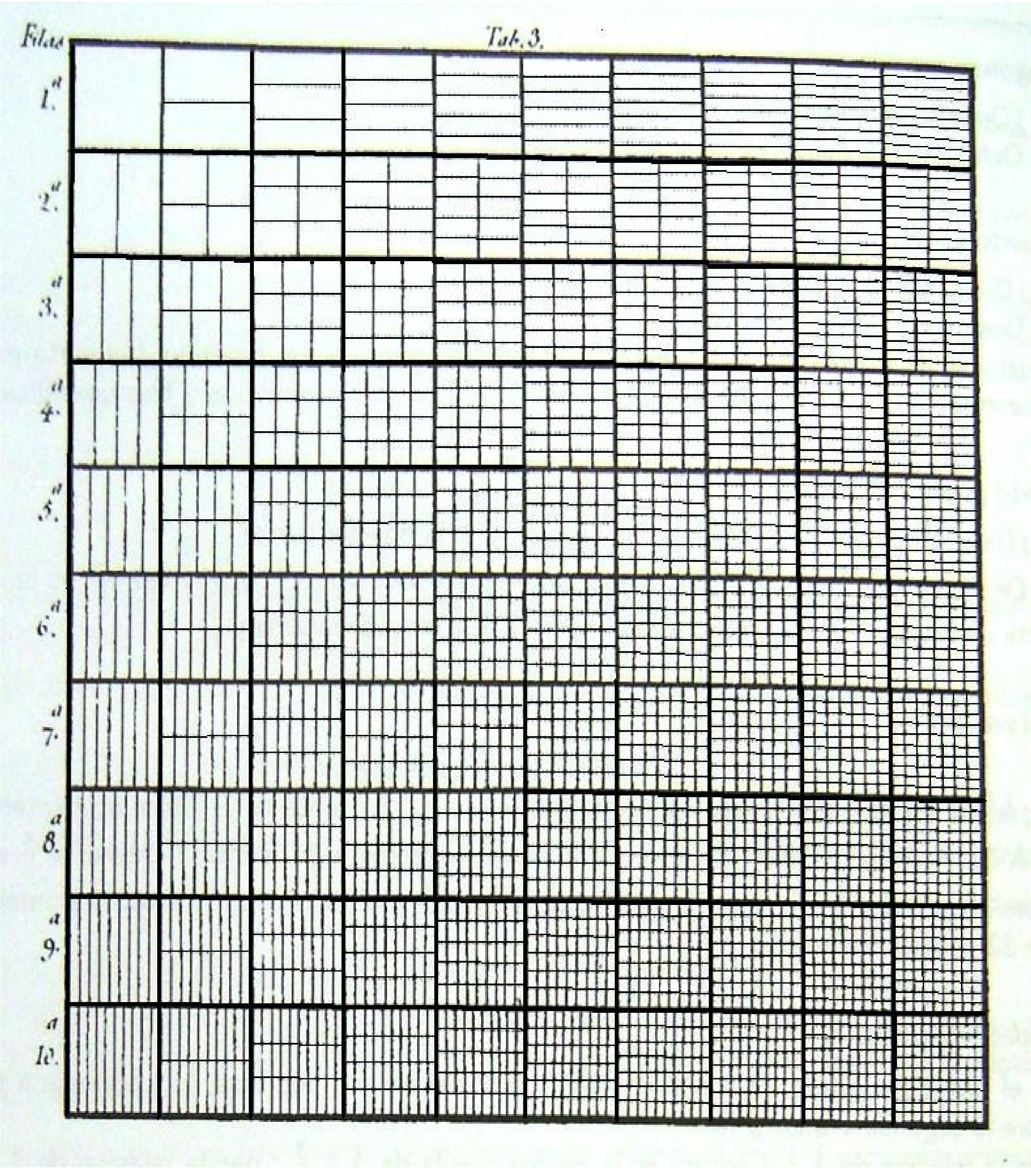
Figure 2
Pestalozzi's Tábua n. 2.



Source: Chavannes, 1809, p. 206.

On this table are held exercises organized in twelve levels. Chavannes (1809) gives an indication about them, although in less detail than in the case of Tábua no. 1. The exercises assume resorting to table row by row, interpreting the various ways of each set of squares and parts of the square. This second table shows the units the student how divisible objects, which several parts form different fractions and sums of parts of units. The exercises in this table are similar to the previous one, only to acquire a much greater extent by considering the fraction of the unit.

Figure 3
Pestalozzi's Tábua n. 3.



Source: Chavannes, 1809, p. 208.

The Tábua n. 3 is an enlargement of the Tábua n. 2. Also, it is listed ten rows of ten square divided by the same way as the vertical lines on Tábua n. 2. In addition to the square splitting into equal parts with horizontal lines of the second column in two parts, the third column in three parts, until the tenth column is divided into ten equal parts. With this table you can find out the relationship of different fractions and reduce them to a common denominator faster and so tangible at the same time.

On this table were held eight different types of exercises. The Pestalozzi's proposed, especially in regard to Tábuas n. 2 and n. 3 was too ambitious for the place that gave the Arithmetic in school, which might explain why these exercises were not spread. According to Gallego (2005), understanding the exercises that Pestalozzi has proposed for the Tábuas n. 2 and n. 3 were too difficult for the most elementary teachers, whom only have known some integer operations.

The Grube's method

After the presentation of the Pestalozzi's ideas, the Augustus G. Grube's thoughts will be introduced. He was a German teacher who in 1842 published the *Guide for calculating the elementary classes, following the principles of a heuristic method*. As Pestalozzi, Grube has considered the intuition as the cornerstone of all learning. However, he has disagreed with the ordering of the contents (Aguayo, 1966).

According to Buisson (1887), the intuitive calculus is a term that denotes a way of teaching the first elements of the calculation. This methodology was created in Germany, spread in Russia, Holland, Sweden and found strong acceptance in the United States. This mode of teaching was called Grube method.

Grube published in 1842, in Berlin, the first edition of his *Leitfaden für das Rechnen in der Elementarschule nach den Grundsätzen einer heuristischen Methode*. This *Essai d'instruction éducative*, as he called it, after provoking heated discussions, has obtained accession of the class of teachers. The Grube's book hits to be in accordance with the new system of weights and measures and it has reached in 1873, in its fifth edition.

The Grube's method states that the students themselves and intuitively make the fundamental operations of elementary calculus. This method aims to make known the numbers: knowing an object, which is not only know your name, but seeing it in all its forms, in all its states, in their various relationships with other objects, comparing with others, following in their transformations, write and measure, compose and decompose at will.

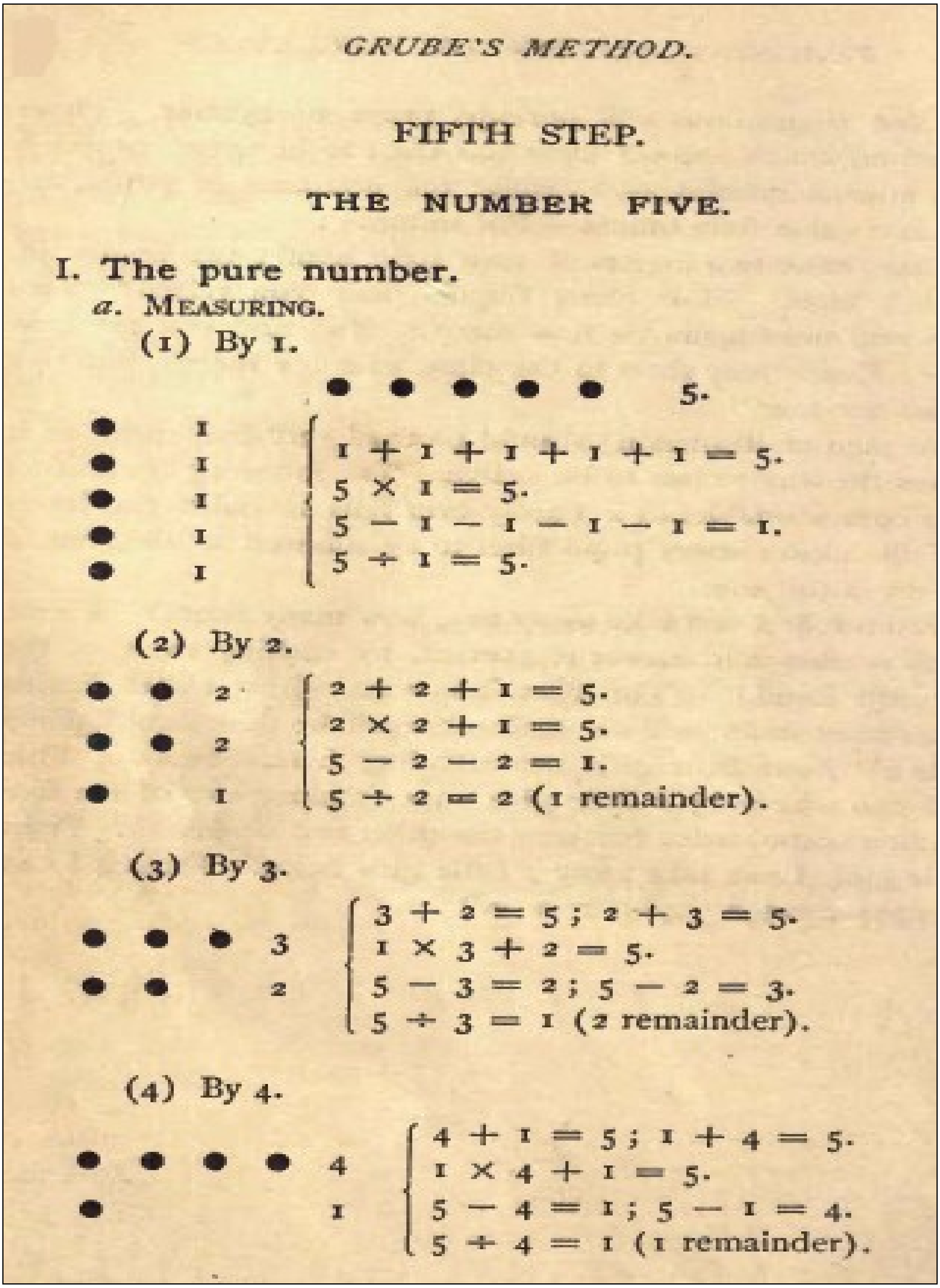
Treating numbers as any objects that become familiar to the intelligence of the students, Grube opposes the old sequence of teaching in learning first successively adding, after the subtraction, followed by multiplication and division. The first old sequence modifications that consisted in eliminating the grouping large numbers, it means, the elementary teaching was divided in the first year of the study the numbers 1 to 10; in the second year of the study the numbers 10 to 100; in the third year to study the numbers 100-1000; and so on, ending with the fourth year of study with the fraction. Grube, however, went beyond this classification. He ruled out the use of large numbers, hundreds and thousands, early in the course, and instead of dividing the teaching of the numbers in the primary level in three or four parts, for instance, 1 to 10, 10 to 100, etc., he has considered each number as a part of itself and it has taught by the following method: it has recommended that the child should learn the relations and operations of addition, subtraction, multiplication and division of each number in a sequence, starting from the number 1, before moving on to the number successor (Soldan , 1878) .

Taking the number 2 as an example, Grube let the child makes all the operations that were possible within the limits of this number, it means, all those who did not use numbers greater than 2 itself, regardless of the operation that was being made. The child should have in mind that:

$$1 + 1 = 2,\ 2 \times 1 = 2,\ 2 - 1 = 1,\ \ 2 \div 1 = 2,\ \text{etc...}$$

The full circle of operations regarding the number two is exhaustively done before the child is submitted to the considerations of the number 3, which is treated in the same way.

Figure 4
Grube's method, 5th step: the number 5.



GRUBE'S METHOD.

FIFTH STEP.

THE NUMBER FIVE.

I. The pure number.

a. MEASURING.

(1) By 1.

5.

1
1
1
1
1

$1 + 1 + 1 + 1 + 1 = 5.$
$5 \times 1 = 5.$
$5 - 1 - 1 - 1 - 1 = 1.$
$5 \div 1 = 5.$

(2) By 2.

2
2
1

$2 + 2 + 1 = 5.$
$2 \times 2 + 1 = 5.$
$5 - 2 - 2 = 1.$
$5 \div 2 = 2$ (1 remainder).

(3) By 3.

3
2

$3 + 2 = 5$; $2 + 3 = 5.$
$1 \times 3 + 2 = 5.$
$5 - 3 = 2$; $5 - 2 = 3.$
$5 \div 3 = 1$ (2 remainder).

(4) By 4.

4
1

$4 + 1 = 5$; $1 + 4 = 5.$
$1 \times 4 + 1 = 5.$
$5 - 4 = 1$; $5 - 1 = 4.$
$5 \div 4 = 1$ (1 remainder).

Source: Soldan, 1878, p.18.

Soldan (1878) indicates the five most important points of the Grube's method of teaching : a) Language : language is the only way by which the teacher will have access to the student 's thinking cause it is not required any record of the calculations made by them. It should require the student a complete answer cause it is only way that the teacher can evaluate as much as the student has learned or not. b) Questions: the teacher should avoid asking too many questions. Students should speak as much alone. c) Individual recitation and together with the class: in order to bring animation class, the answers to the questions should be given alternately individually and then in groups, mainly following the numerical diagram[4]. d) Illustration: each process and each instance should be illustrated through objects that must necessarily be present in class. e) Comparison and measurement: the operation of each stage is to compare and measure each new number with the foregoing, taking place of the relationship or difference quotient, comprising up the four fundamental rules. Associated with this action, beyond the pure numbers called, should make enough examples with numbers applied[5]. f) Writing figures: as the method progresses, students should be able to draw the numerical diagrams.

Intuitive method and numerical diagrams or Cartas de Parker

According to Montagutelli (2000), Francis Wayland Parker (1837-1902) has developed an educational system which was recognized by John Dewey as the "father of progressive education". Coming from a family of educators, Parker has taught since his sixteen years old, and later served in the army during the Civil War. After the war ended, he was principal of a school in Ohio. In 1872 he made a study trip to Europe, and in Germany, became familiar with the pedagogy of Herbart. Presumably this time took note of the Grube's method. In 1875 he returned back to America and became superintendent of schools in Quincy, Massachusetts. At this time, Parker has developed what was called the system Quincy. In an atmosphere free of a rigid discipline imposed in most schools this time, the students read newspapers or texts composed by their teachers. It has been starting from the known, specifically addressed the new notions followed by work group, and practice drawing and music.

He has published five books: *Talks on teaching*[6] (New York, 1883), *The practical teacher* (1884); *Course in arithmetic* (1884); *Talks on pedagogies* (1894) and *how to teach geography* (1885).

Parker has prepared the numerical diagrams based on Grube's method. These numerical diagrams were called the Cartas de Parker and they represent the way to treat the teaching of Arithmetic intuitively. Moreover, they are presented as a reference for the elaboration of mathematics textbooks for the lower levels.

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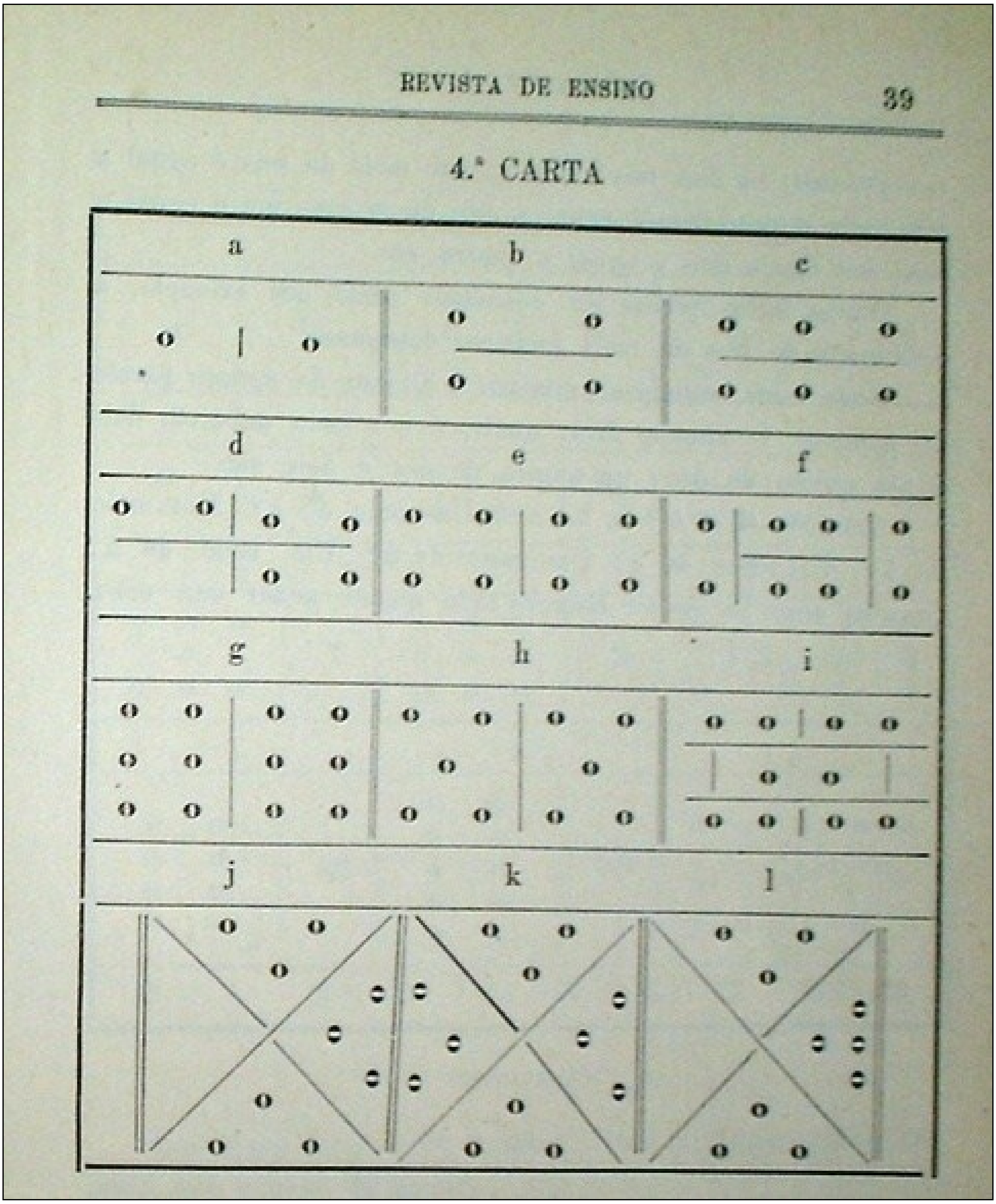
[4] This part of the text refers to what will be seen later as the *Cartas de Parker*.

[5] A pure number or also called an abstract number is one that mentions only the amount. Four, thirty, twelve are examples of pure numbers. On the other hand, if there are explicit units, then this number becomes an applied number or an concrete number. Thirty apples, four trees, three meters, are examples of applied numbers or concrete numbers.

[6] This book was translated to Portuguese by Arnaldo de Oliveira Barreto in 1909 and edited by Livraria Casa Azul.

Figure 5
Carta de Parker n. 4.



Source: Revista de Ensino, 1902, p. 39.

The *Revista do Ensino/SP*[7] has published in several editions around 50 of these numerical diagrams, spreading them in Brazil. For a heuristic process, the teacher questioned the student on the diagram. Example: in the items letters h, i and l are designed to represent the numeral ten. By observing, the student is taken to respond or to make its statements regarding the formation of this number. This is, for example, the letter h it takes two fives to form a ten; in the letter l found the 3 + 3 + 4 to form the ten, as well as the letter i takes five times the two to form a ten (Costa, 2009).

Dewey's influence: the psychology of number

In Brazilian Arithmetic textbooks from early 20th century there are traces of the influence of John Dewey. An example is *Série graduada de matemática elementar* of Barreto (1912). The following is an excerpt of the sixth lesson:

> O centimetro
> (O professor deve utilizar-se dos tornos de um centimetro e de uma fita-metro.) Tome um desses tornos. Que comprimento terá elle? Vamos medir. Que comprimento tem? Nina disse que tem um centimetro de comprimento. Compare o comprimento desse torno com o de outro. Que comprimento terá esse outro? Trace no quadro negro uma linha horizontal de comprimento igual ao do torno. Verifique si está certo. Que comprimento tem a linha? Trace uma linha vertical de um centimetro de comprimento. Verifique. Trace outra vertical de dois centimetros. Verifique. Trace outra horizontal tambem de dois centimetros. Verifique. Outra de um centimetro. Outra de tres. Faça uma linha longa. Meça ahi um comprimento de tres tornos e apague o resto. Que comprimento tem a linha que ficou? Faça outra linha. Meça o comprimento de quatro tornos e apague o excedente. Que comprimento tem a linha que ficou? Numa recta de trescentimetros quantos tornos pode enfileirar ponta com ponta? E numa de dois centimetros? E numa de quatro centimetros? (Barreto, 1912, p. 37)

For Dewey (1895), the number is the result of a reasoning process and not simply a fact sensitive. The mere fact that there is a multiplicity of existing things, or that this multiplicity is present in the eyes and ears does not count towards the consciousness of the number. Although an older child placed in front of five objects and his attention is fixed on these objects, nothing of it will give him the idea of the number. Number is not a property of the objects that can be observed through the use of simple sense. No concept, no clear idea about the set number can enter the consciousness until the thought is ordered to the objects, for instance, related and compared between them somehow.

The origin of the number can be seen with a vague estimate of the dimensions of width, length, weight, compared - related - to the exact value of a unit, and the repetition of which, in space or in time, so does the measure of the set. The measure process is defined in three stages: a) to measure with an indefinite unit. Example: to measure the length of a table with the palm of the hands, or counting by comparing apples to apples unit b) to measure with a unit properly defined. Example: once defined the length of a

[7] *Revista de Ensino* da Associação Beneficente do Professorado Público de São Paulo. São Paulo: Typografia do Diario Official, 1902-1919.

match by a centimeter, how many centimeters have the teacher's desk? c) to measure with a unit that relates to another two different. Example: do not compare volumes and masses, but both relate to a third unit called density (Dewey, 1895).

Final remarks

In an attempt to synthesize as the conceptions of a number, it can be said that the oldest presented conception of numbers in the first forms of teaching shows the number only as the symbol of a group. Arithmetic was a purely utilitarian subject, summarizing all the symbols and all the rules were governed according to numerical manipulation, for example, the written numeral "5" was synonymous with the number five and taught by a severe and deductive process based on imitation and endless repetition. Teachers who share this conception, the oldest in the history of mathematics, did not take into account any educational purpose and they taught Arithmetic by dogmatic processes.

The number can also be considered as a sensory intuition. Pestalozzi was the first who used a psychological basis for conceptualizing the number. For him, the number is more than a symbol, it is a mental image caused by a sensory experience. For example, we could see four books, we could picked four coins up, we could hear four sounds: these all impressions are stored in the brain and the mind, by one to one correspondence, transforms them to the consciousness of four. As one acquires the idea of red, rough and hot, the idea of the number was formed by the experience of certain sensory impressions. This conception is inferred that the number should be taught with concrete means - fingers, peas, pebbles, beads - as Pestalozzi did and the best process for this purpose is the operation count.

On the other hand the number can also be considered as a result of a relative measurement. McLellan and Dewey, in *The psychology of number* (1895), exposed the origin and function of the number by the psychological conception. This theory has at least three contributions to the knowledge about the number: number is not an intuition, but an idea. The number is a relationship and it is formed by the measurement process.

The teaching number for this conception is associated with measures and comparisons. No one sees, touches or hears the number 4. You can see four pencils, four notebooks, four books, because we see the pencils, the notebooks, the books when they are present, but we cannot, however, realize their numbers. When you want to know it, we need to analyze the four groups, it means, to count it, to compare it to the number one or the other to serve as a unit, and then synthesize the units that form into this group. The number is not a mental image, but the interpretation of a sensory experience.

Finally, we highlight the work of Rene Barreto announced in this text, the series of articles arranged in several issues of *Revista de Ensino/SP* about the Cartas de Parker, the Portuguese version of the Parker's book written by Arnaldo de Oliveira Barreto as well Dewey's reference in Brazilian textbooks in Arithmetic, are strong evidence of the influence of these authors in the constitution of the history of the teaching of Arithmetic in Brazil at the turn of the 19th century to the early 20th century.

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