Multi-Sector Thirlwall’s Law
and Technological Progress

Interações entre a Lei de Thirlwall Multissectorial
e o Progresso Tecnológico

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Abstract: In this paper we introduce Thirlwall’s Law in the Pasinetti’s model of Structural Change in order to study uneven development when technological progress is taken into account. The main result of Thirlwall’s analysis is verified here, that is, the growth rate of per capita income of a country depends on the growth rate of its exports. When technical progress is considered this result is still valid but the sectoral rates of technical progress may affect the growth rate of per capita income.

Keywords: Structural economic dynamics. Uneven development. North-South trade. Technical progress. Engel’s Law.

Resumo: Neste artigo deriva-se uma versão multissectorial da Lei de Thirlwall na presença de progresso tecnológico. A principal conclusão do modelo de Thirlwall é obtida aqui, ou seja, a taxa de crescimento da renda per capita depende da taxa de crescimento das exportações, mas esse resultado em sua versão multissectorial é afetado também pelas taxas setoriais de progresso tecnológico.


JEL Classification: O19; F12.

1 Introduction

The relationship between economic growth and international trade has been an important area of study nowadays, specially the connection between exports and growth. In this context, Thirlwall’s law is one of the

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most interesting findings in the Post-Keynesian Economics not only by its simplicity but also for its capacity of explaining the growth experience of a number of countries. In short, the law states that the growth rate of output is given by the ratio of the growth rate of exports to the income elasticity of demand for imports. While the neoclassical model focuses on the supply side to explain the constraints to economic growth, Thirlwall (1979), departing from the Harrod trade multiplier model, focuses on the demand side to explain the growth rates of countries.

A number of papers assessing the empirical validity of Thirlwall’s law in different advanced countries have been published since then and the results are favourable to it.1 Thirlwall (1979) has verified it for a number of developed countries over the post-war period while Thirlwall and Hussain (1982), Moreno (1999), and McCombie and Thirlwall (1994) have shown that the model needs to take into account capital flows to explain the growth experience of developing countries. Atesoglu (1993) has verified its validity for the United States and Léon-Ledesma (1999) by studying the Spanish economic growth has concluded that it has been very close to what is predicted by it.

Bairam (1997) believes that there is an inverse relationship between the elasticities and the relative per-capita output levels with the developing countries having a higher Harrod foreign trade multiplier than the developed countries. As a consequence in times of world-wide economic expansion, other things being equal, the balance of payment accounts of developing countries improve and those of developed countries deteriorate.

In fact there is a large amount of literature on the empirical validity of the Thirlwall’s law but a few has been produced in order to evaluate its meaning in theoretical models. Dutt (2002), by showing how Thirlwall’s analysis can be incorporated into North-South models, is an exception. This author shows that if the income elasticity of imports for the South is higher than that of the North, the world economy will eventually come to an equilibrium in which the South will grow less rapidly than the North, so that the gap between the two regions will grow indefinitely.

Araujo and Lima (2007) has derived the Thirlwall’s Law in a multi-sector set up departing from the Pasinetti’s framework and show that even in the case in which the sectoral elasticities are constant changes in the growth rate of output are possible due to the structural changes that accrue from the evolution of tastes and preferences according to the

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1 López and Cruz (2000, p. 478) pointed out that "All these studies have been carried out under the (implicit) assumption that the real terms of trade or real exchange rate remain constant in the long run". The Journal of Post Keynesian Economics, v. 19, n. 3, Spring 1997, provides a "Minisymposium on Thirlwall’s Law and Economic Growth in an Open-Economy Context".
Engel’s law. In this vein even by considering constant elasticity of demand for imports and exports allows to conclude that there may be structural changes that can explain the variations in the growth rate of output.

From this analysis it is possible to conclude that countries that produce goods with a higher income elasticity of demand can benefit from a higher growth rate of per capita income. If for those commodities in which the country specializes, the growth rate of demand is low the country may face a constrained growth that springs from the Engel’s Law.

In this paper we tackle this point by deriving a multi-sectoral Thirlwall’s Law but in a context of technical progress. In this vein we extend the result obtained by Araujo and Lima (2007) on the relationship between the Thirlwall’s Law and uneven development and we highlight a possible connection on how the constrained BOP growth may affect the process of diffusion/absorption of technological progress in underdeveloped countries. This paper is structured as follows. In the next section we present the Pasinetti’s model of structural in its open version. In section 3, the multi-sectoral Thirlwall’s Law is derived in the presence of technological presence. Section 4 concludes.

2 An Extended Version of Pasinetti’s Model of Structural Change

Araujo and Teixeira (2004) have extended Pasinetti’s analysis to the case of an open economy. Here it is used a version of this model without capital goods that allows us to carry on the Thirlwall’s analysis in a multi-sector economy where productivity and demand varies at particular rates in each of the sectors of two countries: let it A denote the advanced country and U the underdeveloped one. This approach is similar to the one adopted by Dutt (2002), who considers two regions, a poor South and a rich North. Here both economies are assumed to produce $n-1$ consumption goods: one in each vertically integrated sector but with different patterns of production and consumption.

From the point of view of country $U$ the physical and monetary flows of commodities may be summarized by three conditions, that is the condition for full employment of the labour force, the condition for full expenditure of national income and the trade balance equilibrium and the solution for the system of physical and monetary quantities. The full employment condition may be stated as:

$$\sum_{i=1}^{n-1} (a_{in} + \xi a_{in}) a_{ni} = 1$$

(1)
Both \( a_{in} \) and \( a_{in}^* \) stand for the demand coefficients of final commodity \( i \), \( (i = 1, 2, n-1) \). The former refers to domestic and the latter to foreign demand. The production coefficients of consumption goods is \( a_{ni} \). The family sector in country \( A \) is denoted by \( \hat{n} \) and the size of population in both countries is related by the coefficient of proportionality \( \xi \). By adopting exponential dynamic paths for the coefficients appearing in (1), Pasinetti (1981; 1993) shows that even if this condition is fulfilled in an initial time it will present serious difficulties to be fulfilled in the subsequent periods due to the existence of particular rates of technical progress and growth of demand for each of the sectors. In this paper it is followed this approach but here we assume that the unemployment rate is constant through time. The condition for full expenditure of national income is given by:

\[
\sum_{i=1}^{n-1} (a_{in} + a_{in}^*) a_{ni} = 1
\]  

(2)

Where \( a_{in}^* \) is the foreign demand coefficient for commodity \( i \) produced in country \( A \). The equilibrium in trade balance is given by:

\[
\sum_{i=1}^{n-1} (\xi a_{in} - a_{in}^*) a_{ni} = 0
\]  

(3)

An important property of this model is that the trade balance equilibrium may be written not in terms of prices as usual but in terms of labour coefficients: labour coefficients \( a_{ni} \), weight both the export and import demand coefficients for commodities \( i \). Hence, this condition requires that exported commodities expressed in terms of quantities of labour in country \( U \) must be equal to imported commodities also expressed in terms of quantities of labour in \( U \). The solution of the system for physical quantities may be expressed as:

\[
X_i = (a_{in} + \xi a_{in}^*) X_n, \quad i = 1, 2, ..., n-1
\]  

(4)

Thus the physical quantity of each tradable commodity to be produced in country \( U \) will be determined by the sum of the domestic and foreign demands. Being \( p_i \) the price of commodity \( i \) country \( U \), and \( w_i \), is the wage rate (uniform), the set of solution for prices may be expressed as:

\[
p_i = a_{ni} w_U, \quad i = 1, 2, n-1
\]  

(5)
This solution shows that relative quantities of embodied labor continue to regulate relative commodity prices within the boundaries of each country. Let us assume that the medium of exchange (money) is anchored to gold, so that the exchange rate between the two currencies is fixed by the ratio of gold contents of the two monetary units. For the sake of convenience only, let us assume that this ratio is equal to 1. Let us analyse the relationship among international prices considering that the average over-all productivity is ten times greater in \( A \) than in \( U \) but sectoral productivities differ according to a much wider range. The introduction of money in this set up would allow us to perform this analysis taking into consideration variations in the nominal exchange rate that may have an important effect on the structure of the economy.

Araujo and Teixeira (2004) and Pasinetti (1983) have shown that those goods for which differences in productivity are smaller than tenfold will have a lower price in \( U \) that in \( A \). This result shows that even in the case where the average productivity in country \( A \) is higher than in country \( U \), for those sectors in which the productivity in \( U \) is higher than its average productivity, it has comparative advantage in producing these commodities. Those goods for which differences in productivity are greater than tenfold have a lower price in \( A \) than in \( U \). In this case, if international trade is allowed, goods will move between the two countries. People in \( A \) would buy goods of the first type in \( U \), where they are cheaper, and people in \( U \) would buy goods of the second type in \( A \). Country \( U \) would be induced to specialize, and then export, the first type of commodities, while country \( A \) would be induced to specialize, and then exporting, the second type of commodities. These results are important to justify the per capita export and import demand functions that are adopted in the next section.

3 A Multi-Sectoral Thirlwall’s Analysis with Technical Progress

Following Araujo and Teixeira (2004), the dynamic equilibrium in the balance of payment is given by the following expression:

\[
\sum_{i=1}^{n-1} (\xi a_{in} - \hat{a}_{in}) a_{ni} + \sum_{i=1}^{n-1} (\xi a_{in} - \hat{a}_{in}) \hat{a}_{ni} = 0
\] (6)

This expression shows that in order to keep this equilibrium it is necessary that the variation of the balance of payment equilibrium in relation to time is equal to zero. Following Araujo and Lima (2007) let us assume that if \( p_i \leq p_j \), that is if the country \( U \) does not have comparative advantage in producing good \( i \) then the foreign demand for commodity

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\( i \) is equal to zero, that is \( x_{i^n} = 0 \). If \( p_i > p_i \), then let us consider that the foreign demand for commodity \( i \) is given by a typical export function, as the one adopted by Thirlwall (1979). Hence we can summarize this information as follows:

\[
x_{i^n} = \begin{cases} 
\left( \frac{p_i}{p_i} \right)^{\eta_i} Y_A^{\beta_i} & \text{if } p_i > p_i \\
0 & \text{if } p_i \leq p_i 
\end{cases} 
\]  

(7)

Where \( x_{i^n} \) is foreign demand for commodity \( i \), \( \eta_i \) is the price elasticity of demand for export of commodity \( i \) \((\eta_i < 0)\), \( \beta_i \) is the income elasticity of demand for exports and \( Y_A \) is the national income of country \( A \). By dividing both sides of (1) by the population of country \( A \), \( X_A \), we obtain the per capita demand coefficient for foreign demand of commodity \( i \), that is, \( a_{i^n} \):

\[
a_{i^n} = \begin{cases} 
\left( \frac{p_i}{p_i} \right)^{\eta_i} Y_A^{\beta_i} X_A^{1-\beta_i} & \text{if } p_i > p_i \\
0 & \text{if } p_i \leq p_i 
\end{cases} 
\]  

(8)

In the same vein, if the country \( A \) has no comparative advantage in producing good \( i \) the import per capita demand for commodity \( i \) in country \( U \) is equal to zero, that is \( x_{i^n} = 0 \). But if \( p_i > p_i \), then let us consider that the import demand coefficients are given by the usual import demand function, which yields the following functional form for the import demand \( x_{i^n} \):

\[
a_{i^n} = \begin{cases} 
\left( \frac{p_i}{p_i} \right)^{\psi_i} Y_U^{\phi_i} & \text{if } p_i > p_i \\
0 & \text{if } p_i \leq p_i 
\end{cases} 
\]  

(9)

\( \psi_i \) is the price elasticity of demand for import of commodity \( i \) \((\psi_i < 0)\) and \( \phi_i \) is the income elasticity of demand for imports and \( Y_U \) is the real income of country \( U \). By dividing both sides of (9) by the population of country \( U \) we obtain the per capita import coefficient for commodity \( i \), \( a_{i^n} \):
\[ a_i = \begin{cases} 
\left( \frac{p_i}{p_i} \right)^{\psi_i} y_U^\phi X_n^{1-\phi} & \text{if } p_i > p_i \\
0 & \text{if } p_i \leq p_i 
\end{cases} \]  
(10)

If \( p_i > p_i \), we can take natural logarithm in both sides of (8) and differentiate it with respect time. By adopting the following convention:

\[ \hat{p}_i = \sigma_i^U, \quad \frac{\dot{\sigma}_i^U}{p_i} = \sigma_i^A, \quad \frac{\dot{y}_i^A}{y_i^A} = \sigma_y^A, \quad \frac{\dot{X}_n}{X_n} = \hat{\gamma}, \]  
this procedure yields the following growth rate of per capita export demand for commodity \( i \):

\[ \frac{\dot{a}_{i\gamma}}{a_{i\gamma}} = \begin{cases} 
\eta_i \left( \sigma_i^U - \sigma_i^A \right) + \beta_i \sigma_y^A + (1 - \beta_i) \hat{\gamma} & \text{if } p_i > p_i \\
0 & \text{if } p_i \leq p_i 
\end{cases} \]  
(11)

By adopting the same procedure in relation to expression (10) in the case of \( p_i > p_i \) and by adopting the convention that \( \frac{\dot{w}_U}{w_U} = \sigma_w^U \) and \( \frac{\dot{X}_n}{X_n} = g \) we obtain the following growth rate of per capita import demand coefficient for commodity \( i \):

\[ \frac{\dot{a}_{i\gamma}}{a_{i\gamma}} = \begin{cases} 
\psi_i \left( \sigma_i^A - \sigma_i^U \right) + \phi_i \sigma_w^U + (1 - \phi_i) g & \text{if } p_i > p_i \\
0 & \text{if } p_i \leq p_i 
\end{cases} \]  
(12)

Let us assume that \( g = \hat{\gamma} = 0 \), that is the population in both countries remain constant for the sake of simplicity only. Araujo and Lima (2007) assume that \( \sigma_i^U = \sigma_i^A \) that is the rate of change of price of commodity \( i \) is equal in both countries. Thirlwall considers that this hypothesis is justified in terms of the parity power purchase hypothesis. But since we consider technical progress in this version of the paper we have to consider now the possibility that \( \sigma_i^U \neq \sigma_i^A \).

The reason for this specification is twofold: first there is technological progress now and the labor coefficient evolves through time according to the following expression: \( a_{ni}(t) = a_{ni}(0)e^{-\rho t} \). The second reason is that for some authors by excluding relative prices from the demand equations may imply an incorrect model specification [see Hieke (1997)]. The growth rate of technical coefficient in sector \( i \) is given by the following expression:
\[
\frac{\dot{a}_{ni}}{a_{ni}} = -\rho_i
\]

(13)

Since \( p_i = a_{ni} w_U \) and \( \dot{p}_i = a_{ni} \dot{w}_U \), we have that \( \dot{p}_i = a_{ni} \dot{w}_U + a_{ni} w_U \) and \( \dot{p}_i = a_{ni} \dot{w}_A + a_{ni} w_A \). This yields the following growth rates for prices in country \( U \) and \( A \).

\[
\sigma_i^U = \sigma_w^U - \rho_i^U
\]

(14)

\[
\sigma_i^A = \sigma_w^A - \rho_i^A
\]

(15)

There is no reason to assume ex ante that \( \sigma_i^U = \sigma_i^A \). Since the wage rate is the same for all sectors of country \( U \), the per capita income in this country, denoted by \( y_U \), is given by:

\[
y_U = \frac{\sum_{i=1}^{n-1} x_{ni} w_U}{X_n}
\]

(16)

\[
y_A = \frac{\sum_{i=1}^{n-1} x_{ni} w_A}{X_n}
\]

(17)

Where following the Pasinettian convention \( x_{ni} \) is the amount of workers employed in sector \( i \). By assuming that the unemployment rate is constant over time, which is the same as saying that \( \sum_{i=1}^{n-1} x_{ni}/X_n \) is constant over time, we conclude that:

\[
\sigma_y^U = \sigma_w^U
\]

(18)

Where \( \sigma_y^U \) is the growth rate of per capita income in country \( U \). By following the same approach in relation to country \( A \) let us consider that:

\[
\sigma_y^A = \sigma_w^A
\]

(19)
By replacing expressions (14) and (15) into expressions (16) and (17) respectively it yields:

\[
\frac{\dot{a}_{in}}{a_{in}} = \begin{cases} 
(\beta_i + \eta_i)\sigma_y^A - \eta_i(\rho_i^U + \sigma_i^A) & \text{if } p_i > p_i^A \\
0 & \text{if } p_i \leq p_i^A 
\end{cases}
\]  

(20)

\[
\frac{\dot{a}_{in}}{a_{in}} = \begin{cases} 
(\phi_i + \psi_i)\sigma_y^U - \psi_i(\rho_i^A - \sigma_i^U) & \text{if } p_i > p_i^A \\
0 & \text{if } p_i \leq p_i^A 
\end{cases}
\]  

(21)

By replacing expressions (13), (20) and (21) into expression (6) it yields after some algebraic manipulation the following relationship between the growth rate of per capita in countries \(U\) and \(A\).

\[
\sigma_y^U = \frac{\sum_{i=1}^{n_A} \delta_i a_{iA} a_m}{\sum_{i=1}^{n_A} \sum_{i=1}^{n_B} (\eta_i + \beta_i)} \frac{\sum_{i=1}^{n_A} \frac{\delta_i}{a_{iA}} \sum_{i=1}^{n_A} \frac{\delta_i}{a_{iA}} a_{iA} + a_m}{\sum_{i=1}^{n_A} \sum_{i=1}^{n_B} (\eta_i + \beta_i)}
\]

(22)

Expression (22) is a generalization of the disaggregated Thirlwall’s law obtained by Araujo and Lima (2007) when technical progress is taken into account. It shows that the growth rate of per capita income in country \(U\) depends not only on the growth rate of per capita foreign demand but also on technical progress. Note that if \(\rho_i^U = \rho_i^A = 0\) then expression (22) reduces to the version obtained by Araujo and Lima as expected. Technical progress plays an important role in this analysis. Note that if \(\rho_i^U > \rho_i^A\) then even if \(\frac{\sum_{i=1}^{n_A} \delta_i a_{iA} a_m}{\sum_{i=1}^{n_A} \delta_i a_{iA} a_m} < 1\) the growth rate of per capita in country \(U\) may approach the growth rate of per capita in country \(A\). But the usual assumption is the reverse, that is, \(\rho_i^U < \rho_i^A\). In this case this will pose more difficulties for the underdeveloped country to grow faster or equal to the advanced one.

This result is not surprising since technical progress is one of the engines of economic growth and it allows highlighting another possible connection between growth and technological progress raised by Kaldor (1957) through his formulation a function of technological progress. According to Ocampa (2005, p. 25), “[…] this link between productivity and production growth […] may also be called the Kaldor-Verdoon function” and “[if] the casual links are correct, then productivity improvements are largely the result of dynamic economic growth, a causal link
that is just the opposite of that assumed by neoclassical growth theory since Solow (1956, 2000)."

Hence we can say that the Thirlwall’s analysis is enriched when technical progress is introduced in the model but his initial insight is still valid. A favourable specialization pattern affects economic growth by increasing not only the growth rates but also on maintaining high levels of productivity growth which also has a positive impact on the growth rate of per capita income. A possible conclusion is that poor regions need to induce structural changes in their economies that encourage the expansion of export oriented manufacturing industries, producing commodities with higher elasticities of demand than those for primary products.

An important point to be considered in relation to the derivation performed in this article is that it has been carried out under the assumption that the real terms of trade or real exchange rate remain constant in the long run (LÓPEZ; CRUZ, 2000, p. 478) meaning that relative prices stay invariant and that they do not play an important role in the long run. This view is supported by authors such as Alonso and Garcimartín (1998) who consider that relative prices do not play any role for two reasons: First is related to the fact that empirical studies have been showing that Purchase Power Parity\(^2\) - PPP hereafter - holds in the long run. The second suggests that the price elasticities are very small.

However even in the case in which PPP holds variations in the exchange rate play an important role in structural change since the competitiveness of the sectors relies heavily on the nominal exchange rate. Following this rationale and considering a multi-sectoral version of the Thirlwall’s it is possible to show that the growth rate of a country is strongly affected by permanent movements in the nominal exchange rate. An appreciation of the exchange rate reduces the competitiveness of the national industries slowing down the growth and investment. Although this is an important mechanism it was not taken into account in the present paper.

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\(^2\) According to the PPP hypothesis the exchange rate between two currencies will move in line with relative price levels in the two economies. For commodities in which the Law of one Price holds the PPP would hold continuously. In this paper we intend to show that the real exchange role does not play a passive role as assumed in the models of the Thirlwall’s tradition.
4 Concluding Remarks

In this paper the Thirlwall’s analysis is carried out by using an extended version of Pasinetti’s model of structural change that considers foreign commerce and technological progress. The main result of the Thirlwall’s analysis was verified here, that is, the growth rate of per capita income of a country depends on the growth rate of exportations, captured here by the growth rate of per capita foreign demand. When technical progress is taken into account this result is still valid but the sectoral rates of technical progress may affect the growth rate of per capita income of the underdeveloped country. All results obtained here have shown that the growth rate of the underdeveloped countries is affected by its capacity of exporting, which by its turn is negatively affected by the low income elasticity of primary products.

However it is important to consider that the determination of the growth rates is due not only to elasticities but also the weigh that these goods have in the economy. By neglecting the exports of primary goods for instance a developing country may damage the ability of taking advantage of the share that these commodities has in the exports which may compensate their small elasticity of demand.3 One of the consequences of neglecting the exports of commodities is that the trade deficit may not drop fast enough to compensate for declining commodity trade surpluses.

In order to developing countries narrow the income gaps with richer ones for instance they need to create leading industrial sectors, along with related technological and social capabilities taking into account the structural change that accompanies economic growth. However, such structural changes rarely occur as a smooth or harmonious process. Indeed they pose new and difficult economic challenges for developing economies. In particular, the absorption of technical progress developed in advanced nations is a daunting task for developing country since the consequence of adopting new technologies may be not only increases in productivity but structural unemployment. Besides, this paper acknowledges that the efficiency of any development strategy is not independent of current as well as prospective world economic performance as a whole.

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3 Data shows that the good performance of the Brazilian exports have is due to an increased of China demand for commodities due to the growth experience in the latter, which allows a number of consumers to take part in the consumer market. The small elasticity of demand of these commodities have been compensated by the share they have on the Brazilian exports.
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