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Instituto de Geociências, Universidade Federal do Rio Grande do Sul, Porto Alegre, RS, Brasil

Aerotriangulation in Microcomputer

FRANCISCO HUMBERTO SIMÕES MAGRO¹ E JOSÉ BITTENCOURT DE ANDRADE²

¹ Instituto de Geociências, UFRGS, Caixa Postal 15001, CEP 91501-970, Porto Alegre, RS, Brasil

² Curso de Pós-Graduação em Ciências Geodésicas, UFPR, Caixa Postal 19011, CEP 81530-900, Curitiba, PR, Brasil

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Abstract — This paper presents a brief description of the methods utilized to correct systematic errors of image space coordinates of analytical phototriangulation. An explanation of the adjustment of the photogrammetric block is also given, presenting the algorithms, for the technique used in the solution of normal equations utilizing the iterative method of successive overrelaxation. Finally, the results of some experiences are presented analysing processing time to solve the system of equations with different microcomputer models.

Resumo — Neste trabalho é feita uma descrição sucinta, informando os métodos utilizados para corrigir os erros sistemáticos, das coordenadas do espaço imagem de uma fototriangulação analítica. Além disso, dá-se uma explanação, apresentando os algoritmos, do ajustamento do bloco fotogramétrico aprofundando quanto à técnica utilizada para a solução das equações normais com o método iterativo das sobre-relaxações. Por último, apresentam-se os resultados de algumas experiências efetuadas onde se analisa a duração de processamento na resolução do sistema de equações em diferentes modelos de microcomputadores.

INTRODUCTION

With analytical phototriangulation it is possible to obtain the coordinates of the object space from the coordinates of the image space. For this transformation a system of computational programs was developed which used the bundle adjustment method to obtain the coordinates of the object space from the coordinates of the image space. This procedure basically consists of the following stages:

- a) correction of systematic errors such as, film deformation, translation of the principal point, lens distortion and photogrammetric refraction applied to the image space coordinates;
- b) computation of the approximate parameters, and
- c) adjustment of analytical phototriangulation using the least squares method with a solution of the normal equations system using the iterative method of successive overrelaxation.

When there are photogrammetric blocks with over 50 aerial photographs, items a and b take little time (a few seconds), but item c has proved to be a lengthy operation (several hours) when the computational program system begins operating.

The purpose of this study is to perform several experiments, using the PC XT, AT286 and AT386 model computers to find out how long it takes to adjust a photogrammetric block for different sizes.

MATERIAL AND METHODS

Initial considerations

Analytical aerotriangulation by bundle adjustment was developed with a pre-refinement of the coordinates of the image space points. In this case the mathematical model was the colinearity equation with weight

constraints, and the observations must be cleared of systematic errors. In this way it is admitted that the knowledge of causes of systematic errors and their correction were sufficient to consider residual errors accidental. With the removal of systematic errors from the image space coordinates, the image space point, the perspective center of the camera and the object space point begin to verify colinearity condition. Thus, when the colinearity equation is applied to the coordinates which define those three points, the colinearity condition will be verified or be very close to physical reality. The adjustment of aerotriangulation was performed according to the method of least squares with the solution of the normal equations system using the iterative method of successive overrelaxation.

Systematic errors

The systematic errors handled were film deformation, non coincidence between the principal point and fiducial center, lens distortion and photogrammetric refraction.

Correction of systematic errors resulting from film deformation, due to changes in temperature and humidity, variable tension and film shrinking was performed during the transformation of arbitrary instrumental coordinates into fiducial coordinates. Depending on the amount of fiducial marks, different mathematical models are used. In this study, for the general case of aerial photographs with 4 fiducial marks, the mathematical model of affine transformation was developed, as can be seen in Slama *et al.* (1980) and Merchant (1978).

The correction of the systematic error of non-coincidence of the principal point with the fiducial center is performed by translation, transforming the photographic system into a fiducial system, as can be seen in Slama *et al.* (1980).

The correction of systematic errors in lens distortion, i.e., symmetrical radial distortion and decentered distortion, followed the mathematical models developed by Conrady (1919) and Brown (1966).

In correcting systematic errors of objective lens distortion, symmetrical radial distortion and decentered distortion were taken into account.

The correction of the systematic error of symmetrical radial distortion followed the mathematical model developed by Conrady in 1919, and presented in Andrade & Olivas (1981). The correction of the systematic error of the decentered distortion followed the mathematical model of Conrady-Brown as seen in Merchant (1979).

The correction of photogrammetric refraction was performed according to the mathematical model developed by Andrade (1977).

Block adjustment

For the simultaneous adjustment of a photogrammetric block the parametric method was used, in which the image coordinates had already been corrected for systematic errors.

The observation equations may be mathematically expressed as follows:

$$\begin{aligned} x &= f \frac{m_{11}(X - X^c) + m_{12}(Y - Y^c) + m_{13}(Z - Z^c)}{m_{31}(X - X^c) + m_{32}(Y - Y^c) + m_{33}(Z - Z^c)} \\ y &= f \frac{m_{21}(X - X^c) + m_{22}(Y - Y^c) + m_{23}(Z - Z^c)}{m_{31}(X - X^c) + m_{32}(Y - Y^c) + m_{33}(Z - Z^c)} \end{aligned} \quad (1)$$

where

(x, y) represent the rectangular coordinates of a point in image space whose systematic errors have been removed;

(X, Y, Z) represent the rectangular coordinates of a point in the object space system;

(X^c, Y^c, Z^c) represent the rectangular coordinates of the perspective center of the camera in the object space system;

f represents the calibrated focal distance or the camera constant; and

$m_{11} \dots m_{33}$ represent the elements of the rotation matrix of the image system, as related to the object space system.

These equations allow the transformation of rectangular coordinates of a point in the object space into the coordinates of the image space, imposing the collinearity condition on the image space point, the perspective center of the camera and the object space point.

In fitting a block using the bundle adjustment, there are seven degrees of freedom, i. e., one from the scale factor, three from the translations and three from rotations. These seven degrees of freedom may be raised by introducing ground control points which will allow the definition of a coordinates system.

The ground control points, in this study, are considered those which were obtained in geodetic field surveys and by kinematic tracking during flight to define the coordinates of the perspective center of the camera, at the time aerial photos are taken.

In the first case, the ground control points must be easily identifiable in the aerial photos and may be planimetric, altimetric or planialtimetric. The positional constraint equations which eliminate the seven degrees of freedom and, consequently, remove the uniqueness of the matrix of the coefficients of normal equations when being fixed to the local coordinates of the support points, take on the following form:

$$\begin{aligned} X_b - X_a &= v_i \\ Y_b - Y_a &= v_i \\ Z_b - Z_a &= v_i \end{aligned} \quad (2)$$

where

(X_b, Y_b, Z_b) represent the known coordinates of a generic point of the image space in the local system,

(X_a, Y_a, Z_a) represent the adjusted coordinates of a point of the image space, and

v_i represent the residues of each constraint equation.

In the second case, the ground control points are the coordinates of the perspective center of the camera at the time the aerial photo is taken. The positional restriction equations which eliminate the seven degrees of freedom, and, consequently, remove the uniqueness of the matrix of the coefficients of normal equations when fixing the local coordinates of the ground control points as follows:

$$\begin{aligned} X_b^c - Y_a^c &= v_i \\ Y_b^c - Y_a^c &= v_i \\ Z_b^c - Z_a^c &= v_i \end{aligned} \quad (3)$$

where

(X_b^c, Y_b^c, Z_b^c) represent the known coordinates of the perspective center of the camera of the image space in the local system.

(X_a^c, Y_a^c, Z_a^c) represent the adjusted coordinates of the perspective center of the camera of the image space, and

v_i represent the residues of each constraint equation.

The simultaneous adjustment of a block using the least squares method of a bundle block from all the stations exposed to all points of the image space allows the simultaneous recovery of the guiding elements of all photos and the adjustment of the object space points.

In this sense, when observed image space points and object space ground control points are available, which have been observed in the image space, observation equations and positional restriction equations may be generated. The total number of equations in

the simultaneous adjustment of a block will be given by the expression

$$m = EO + EIP \quad (4)$$

where

EO represents the number of observation equations, and EIP represents the number of positional restrictions.

The number of parameters can be found with the help of expression

$$n = 6 NF + 3 NPT \quad (5)$$

where

NF represents the number of pictures in the block and NPT represents the number of different points in the object space.

The colinearity equations, defined by the expression (1) are of the observation equation type, which lead to an adjustment using the parametric method, with the mathematical model

$$L = F(X) \quad (6)$$

The quadratic form $V'PV$, when minimized, leads to the normal equations defined by the matrix expression

$$NX = U \quad (7)$$

where

$N = A'PA$ represents the matrix of the coefficients of normal equations

$U = A'PL$ represents a column vector with n elements which are the absolute terms of the normal equations, and

X represents the vector with n elements which are the corrections to be inserted in the approximate parameters.

Matrix N , whose elements represent the coefficients of normal equations is a square symmetrical, sparse matrix. Its size is equal to the number of system parameters, i. e., the outer orientation elements and the object space coordinates. Using the features of matrix N , the nonzero elements of the upper triangular were stored in a vector. To know how many nonzero elements are in a matrix N , in its upper or lower triangular including the main diagonal, the following expression may be used:

$$ENN = 21 NF + 6 NPO + 18 NPI \quad (8)$$

where

NF represents the number of photos in the block, NPO represents the number of points in the objects space,

NPI represents the number of points in the image space and

ENN represents the nonzero elements.

Solution of the normal equations system

Having solved this system of normal equations the

adjusted parameters may be calculated. Since the values of the approximated parameter may not be so good, and the observation equations are not linear, it is necessary to use an iterative adjustment process to obtain more likely values from the unknowns. Equation

$$X_a = X_o + X \quad (9)$$

generates a first value adjusted to the parameters which must be used as approximate values during the next iterative step.

The iterative process must be interrupted when the convergence criteria are reached.

As a photogrammetric block increases, so does its number of variables. The solution of a system of equations, with few variables, may be an extremely easy task. However, if the photogrammetric block has a large number of variables, which is usually the case, the operation to solve the system of linear equations may go beyond the storage capacity of available computers.

Among the iterative methods to solve linear equation systems used in geodetic sciences, successive overrelaxation was chosen

$$X_i = w (X_{GS} - X_{i-1}) + X_{i-1} \quad (10)$$

where

X_{GS} represents the values calculated for the block by the Gauss-Seidel method

X_{i-1} represents an approximation of the solution vector, and

w the relaxation factor.

This relaxation factor should be chosen so as to render the process convergence as rapid as possible. For $w = 1$ therefore, the Gauss-Seidel iterative method exists, a specific case of the successive overrelaxation method. In Young & Gregory (1973), theorems can show that it should not remain outside the interval

$$0 < w < 2 \quad (11)$$

and it is also stated that if it lies within this interval, and matrix N is true and positively defined, the successive overrelaxation method will always converge. On the other hand, studies by Carlson & Haljala (1974), have recommended that for stable blocks with a moderate geometric structure, the value of w should be between 1.8 and 1.9, whereas for weak blocks it should range between 1.9 and 1.94.

Convergence criteria

In the method developed during this study, an iterative process was applied, the resolution of normal equations systems, within another iterative process, the parametric method. Therefore it became necessary to establish convergence criteria which would end the iterative processes during the processing of computational programs.

During the iterative process to solve the system of equations, two levels of convergence criteria occurred:

- a) one within the aerotriangulation program which interrupts processing when the number of iterations is higher than three times the number of parameters.
- b) the other can be established by the user, for which 0,0001 rd is recommended for the corrections of the Eulerian angles and 0.005m for the corrections of the object space point coordinates which was used successfully for the experiments performed.

The iterative process of adjustment ends when the corrections of the adjusted coordinates of the object space are less than 10% of the mean quadratic error of the observations, considering the scale as related to the object space. Thus, this convergence criterion may be expressed using the formula.

$$CV = 0,10 \text{ EMQ} / 1.000.000 \times H / f \quad (12)$$

where

CV represents the convergence criterion expressed in meters

EMQ represents the mean quadratic error observations

H represents the mean flight height expressed in meters

f represents the focal length of the camera expressed in meters and

1000000 represents a constant which transforms the dimension micra of EMQ into meters.

Processing time

The processing time of the programs which correct the systematic errors and calculate the approximate parameters is so insignificant that no one bothered to create an expression which would allow the calculation of computational time. However, to adjust the block, specifically regarding the resolution of the system of normal equations, as can be seen in Magro (1990), we have the empirical expression.

$$T = \left(\sum_{i=1}^k l_i \right) t n \text{ ENN} \quad (13)$$

where

l_i represents the total number of iterative steps in the resolution of the system of normal equations using the iterative method of successive overrelaxation;

n represents the number of parameters of the photogrammetric block;

ENN represents the number of nonzero elements of the upper (or lower) triangular, of the normal equation;

k represents the total number of iterative steps of adjustment in the parametric model;

t represents time expressed in hours per parameter and per nonzero element as related to a given computer; and

T represents the approximate value of hours required to process the block.

Quality control

In adjusting the photogrammetric blocks, two types of models may be considered, i. e. functional and random.

As to the functional model, its shape was seen when the observation and positional constraint equations were established, which show the adjusted observation as a function of the unknowns or adjusted parameters symbolically represented by

$$L_a = F(X_a) \quad (14)$$

In the random model, for instance, Mikhail (1976) defines it as being the sum total of assumptions of statistical properties of the variables involved. Thus, it is possible to analyse the results of adjustments statistically when the random model is used in an unbiased manner and then accept or improve:

- a) the mathematical model;
- b) the rough observations; and
- c) the compatibility of variances of observations with the required precision of results.

Program system

A computational program system in FORTRAN language was developed by the authors. This program system consists of three programs which, essentially, do the following:

- a) correct systematic errors;
- b) calculate approximate parameters; and
- c) adjust the block.

These programs were installed and compiled by the Microsoft compiler, version 4.0 in the following equipment:

- a) XT microcomputer with 640 kbytes RAM (Random Access Memory) memory, a hard disk with a capacity for 20 mbytes and an arithmetic coprocessor 8087/2.
- b) AT286 microcomputers with a RAM memory, a hard disk with a capacity for 40 mbytes and an arithmetic coprocessor 80287.
- c) AT386 microcomputer without an arithmetic coprocessor.

RESULTS AND DISCUSSION

For the purpose of analysing processing time in the different microcomputer models previously mentioned, an adjustment of a photogrammetric block with 4 photos per band was made. This photogrammetric block contained 240 observations referring to 120 points in the image space, regularly distributed throughout the 16 photos. In the object space there were 28 points and also the relative orientation elements referring to the 16 points, generating 104 parameters.

The program which performs aerotriangulation has a subroutine which, when called, records time and date installed in the operational system. During aerotriangulation, whenever the resolution of the system of normal equations began and ended during the successive over relaxation method, that subroutine generated values. These values were recorded in a data file which enabled the computation of the results of t

in the expression (13) for each type of microcomputer used in this experiment, as can be seen in Table 1.

PC	t (h)
XT	$3,91 \times 10^{-8}$
AT 286	$6,74 \times 10^{-9}$
AT 386	$4,44 \times 10^{-9}$

Table 1 — Time consumed by parameter and nonzero element.

This photogrammetric block used in the experiment with the previously described convergence criteria, processed in the iterative method of successive overrelaxations method 134, 47 and 29 times respectively for the 1, 2 and 3 iterative steps in the parametric method, making up a total of 210 iterative steps for 120 photos. Knowing the value of t in the expression (13) it is possible to compute the time required for the adjustment of a photogrammetric block in one of those microcomputer models. Thus, using the expression (13) and the respective value of t and the total number of iterations, it is possible to obtain the time needed to solve the system of normal equations using the iterative method of successive overrelaxations, as can be seen in Table 2.

PC	Time
XT	4h 32min 34s
AT 286	0h 46min 59s
AT 386	0h 30min 57s

Table 2 — Times resulting from the block for the different microcomputer models used in the test.

To analyse the potentials of the program in terms of time consumed, a photogrammetric block with 100 photos can be idealized, 210 object space points which will result in 1230 parameters and 18480 nonzero elements in the matrix of the normal equation coefficients. If this block were aerotriangulated in these three microcomputers and the total number of iterations were around 100, the time consumed, computed using expression (13) may be seen in Table 3.

Computer	Time
XT	177h 45min 6s
AT 286	30h 38min 26s
AT 386	20h 11min 5s

Table 3 - Time consumed for a 100 photo block.

The results of Table 3 show that for this example block and using this program, processing time is very long when the XT microcomputer is used. However, for microcomputer AT 386, the time is reduced to 10h 11min 5s. Although 20 hours is little as compared with 177 hours, it is a long time, all the same.

CONCLUSIONS

Considering the results obtained and the experiments performed, the following may be concluded:

- The method to solve the normal equations system storing nonzero elements in a vector will take so much the longer as the number of parameters and/or nonzero elements increase.
- For blocks with more than 10 aerial photographs, AT 386 microcomputers should be used.
- For blocks with more than 100 aerial photos it seems that some new thinking should be done about the method, to see whether the benefit/ cost ratio makes it worthwhile to use an AT 386.

Taking these conclusions into account, the authors are developing a combination of techniques using partition and the successive over-relaxations method.

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